

Lecture Recap

(OLG with Money in the Utility Function)



Lifecycle Model with Money



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- ▶ Money demand due to utility maximisation. Money supply initially M_1 and can grow.



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