Recap: Lifecycle Model (OLG with Government)

Government

- Government sources its funds through taxes (T_t) and borrowing from the young (B_t).

Government

- Government sources its funds through taxes (T_t) and borrowing from the young (B_t).
- Government uses its funds on consumption (G_t), capital formation (K_t^g), transfers/subsidy, or debt repayment (B_{t+1}).

Government

- Government sources its funds through taxes (T_t) and borrowing from the young (B_t).
- Government uses its funds on consumption (G_t), capital formation (K_t^g), transfers/subsidy, or debt repayment (B_{t+1}).
- Government budget constraint (GBC) for each period t:

$$G_t + K_{t+1}^g + (1 + r_t)B_t = T_t + B_{t+1}$$

Government

- Government sources its funds through taxes (T_t) and borrowing from the young (B_t).
- Government uses its funds on consumption (G_t), capital formation (K_t^g), transfers/subsidy, or debt repayment (B_{t+1}).
- Government budget constraint (GBC) for each period t:

$$G_t + K_{t+1}^g + (1 + r_t)B_t = T_t + B_{t+1}$$

- Government Inter-temporal Budget Constraint (GIBC) can be written as:

$$B_t = \frac{T_t - G_t}{R_t} + \frac{T_{t+1} - G_{t+1}}{R_t R_{t+1}} + ... + \lim_{s \to \infty} \frac{B_{t+s}}{R_t R_{t+1} ... R_{t+s}}$$

Government

- Government sources its funds through taxes (T_t) and borrowing from the young (B_t).
- Government uses its funds on consumption (G_t), capital formation (K_t^g), transfers/subsidy, or debt repayment (B_{t+1}).
- Government budget constraint (GBC) for each period t:

$$G_t + K_{t+1}^g + (1 + r_t)B_t = T_t + B_{t+1}$$

- Government Inter-temporal Budget Constraint (GIBC) can be written as:

$$B_t = \frac{T_t - G_t}{R_t} + \frac{T_{t+1} - G_{t+1}}{R_t R_{t+1}} + ... + \lim_{s \to \infty} \frac{B_{t+s}}{R_t R_{t+1} ... R_{t+s}}$$

- Transversality condition ensures that the government's debt is sustainable.

$$\lim_{s \to \infty} \frac{B_{t+s}}{R_t R_{t+1} ... R_{t+s}} = 0$$

▶ Assume that $\alpha = 0$, i.e., individuals do not value consumption when young.

Fiscal Policy in OLG Model

▶ Assume that $\alpha = 0$, i.e., individuals do not value consumption when young.

▶ Budget constraint when young:

$$a_{t+1} + b_{t+1} = w_t - \tau_{yt}$$

Fiscal Policy in OLG Model

▶ Assume that $\alpha = 0$, i.e., individuals do not value consumption when young.

▶ Budget constraint when young:

$$a_{t+1} + b_{t+1} = w_t - \tau_{yt}$$

▶ Budget constraint when old:

$$c_{ot+1} = (1 + r_{t+1})(a_{t+1} + b_{t+1}) - \tau_{ot+1}$$

▶ Assume that $\alpha = 0$, i.e., individuals do not value consumption when young.

▶ Budget constraint when young:

$$a_{t+1} + b_{t+1} = w_t - \tau_{yt}$$

▶ Budget constraint when old:

$$c_{ot+1} = (1 + r_{t+1})(a_{t+1} + b_{t+1}) - \tau_{ot+1}$$

▶ Lifetime budget constraint:

$$\frac{c_{ot+1}}{1 + r_{t+1}} = w_t - \left(\tau_{yt} + \frac{\tau_{ot+1}}{1 + r_{t+1}} \right)$$

Fiscal Policy in OLG Model

▶ Assume that $\alpha = 0$, i.e., individuals do not value consumption when young.

▶ Budget constraint when young:

$$a_{t+1} + b_{t+1} = w_t - \tau_{yt}$$

▶ Budget constraint when old:

$$c_{ot+1} = (1 + r_{t+1})(a_{t+1} + b_{t+1}) - \tau_{ot+1}$$

▶ Lifetime budget constraint:

$$\frac{c_{ot+1}}{1 + r_{t+1}} = w_t - \left(\tau_{yt} + \frac{\tau_{ot+1}}{1 + r_{t+1}} \right)$$

▶ Generational account $\hat{\tau}_t = \tau_{yt} + \frac{\tau_{ot+1}}{1 + r_{t+1}}$.

▶ Firm profit maximisation:

$$w_t = A(1 - \beta)k_t^{\beta} \qquad r_t = A\beta k_t^{\beta - 1}$$

▶ Firm profit maximisation:

$$w_t = A(1 - \beta)k_t^{\beta} \qquad r_t = A\beta k_t^{\beta-1}$$

▶ Transition equation for capital:

$$k_{t+1} = a_{t+1} + k_{t+1}^g = w_t - \tau_{yt} - b_{t+1} + k_{t+1}^g$$

Key Conditions

▶ Firm profit maximisation:

$$w_t = A(1 - \beta)k_t^{\beta} \qquad r_t = A\beta k_t^{\beta-1}$$

▶ Transition equation for capital:

$$k_{t+1} = a_{t+1} + k_{t+1}^g = w_t - \tau_{yt} - b_{t+1} + k_{t+1}^g$$

▶ Market clearing:

$$
\begin{aligned}
\text{Labour:} \quad & L_t = N \\
\text{Capital:} \quad & K_t = Na_t + K_t^g \\
\text{Bonds:} \quad & B_t = Nb_t
\end{aligned}
$$