

Lecture Recap

(Solow Growth Model)



Lecture Recap

- ▶ Production Function: $Y = F(K, AN)$.



Lecture Recap

- ▶ Production Function: $Y = F(K, AN)$.
- ▶ Output per effective worker: $y \equiv Y/AN = \frac{F(K, AN)}{AN} = F\left(\frac{K}{AN}, 1\right) = f(k)$.



Lecture Recap

- ▶ Production Function: $Y = F(K, AN)$.
- ▶ Output per effective worker: $y \equiv Y/AN = \frac{F(K, AN)}{AN} = F\left(\frac{K}{AN}, 1\right) = f(k)$.
- ▶ Assume that A and N grow at constant rates g_A and g_N respectively.



Lecture Recap

- ▶ Production Function: $Y = F(K, AN)$.
- ▶ Output per effective worker: $y \equiv Y/AN = \frac{F(K, AN)}{AN} = F\left(\frac{K}{AN}, 1\right) = f(k)$.
- ▶ Assume that A and N grow at constant rates g_A and g_N respectively.
- ▶ Then, to keep K/AN constant, K must grow at the same rate as AN .



Lecture Recap

- ▶ Production Function: $Y = F(K, AN)$.
- ▶ Output per effective worker: $y \equiv Y/AN = \frac{F(K, AN)}{AN} = F\left(\frac{K}{AN}, 1\right) = f(k)$.
- ▶ Assume that A and N grow at constant rates g_A and g_N respectively.
- ▶ Then, to keep K/AN constant, K must grow at the same rate as AN .
- ▶ Capital Accumulation Equation: $K_{t+1} - K_t = sY_t - \delta K_t$.

Lecture Recap

- ▶ Production Function: $Y = F(K, AN)$.
- ▶ Output per effective worker: $y \equiv Y/AN = \frac{F(K, AN)}{AN} = F\left(\frac{K}{AN}, 1\right) = f(k)$.
- ▶ Assume that A and N grow at constant rates g_A and g_N respectively.
- ▶ Then, to keep K/AN constant, K must grow at the same rate as AN .
- ▶ Capital Accumulation Equation: $K_{t+1} - K_t = sY_t - \delta K_t$.
- ▶ Capital per effective worker accumulation equation:

$$\frac{K_{t+1}}{A_t N_t} - \frac{K_t}{A_t N_t} = s \frac{Y_t}{A_t N_t} - \delta \frac{K_t}{A_t N_t}$$

Lecture Recap

- ▶ Production Function: $Y = F(K, AN)$.
- ▶ Output per effective worker: $y \equiv Y/AN = \frac{F(K, AN)}{AN} = F\left(\frac{K}{AN}, 1\right) = f(k)$.
- ▶ Assume that A and N grow at constant rates g_A and g_N respectively.
- ▶ Then, to keep K/AN constant, K must grow at the same rate as AN .
- ▶ Capital Accumulation Equation: $K_{t+1} - K_t = sY_t - \delta K_t$.
- ▶ Capital per effective worker accumulation equation:

$$\frac{K_{t+1}}{A_{t+1}N_{t+1}} - \frac{K_t}{A_tN_t} = s\frac{Y_t}{A_tN_t} - \delta\frac{K_t}{A_tN_t} \implies \frac{K_{t+1}}{A_{t+1}N_{t+1}} \cdot \frac{A_{t+1}N_{t+1}}{A_tN_t} - k_t = sy_t - \delta k_t$$

Lecture Recap

- ▶ Production Function: $Y = F(K, AN)$.
- ▶ Output per effective worker: $y \equiv Y/AN = \frac{F(K, AN)}{AN} = F\left(\frac{K}{AN}, 1\right) = f(k)$.
- ▶ Assume that A and N grow at constant rates g_A and g_N respectively.
- ▶ Then, to keep K/AN constant, K must grow at the same rate as AN .
- ▶ Capital Accumulation Equation: $K_{t+1} - K_t = sY_t - \delta K_t$.
- ▶ Capital per effective worker accumulation equation:

$$\begin{aligned}\frac{K_{t+1}}{A_{t+1}N_{t+1}} - \frac{K_t}{A_tN_t} &= s\frac{Y_t}{A_tN_t} - \delta\frac{K_t}{A_tN_t} \implies \frac{K_{t+1}}{A_{t+1}N_{t+1}} \cdot \frac{A_{t+1}N_{t+1}}{A_tN_t} - k_t = sy_t - \delta k_t \\ &\implies (1 + g_A + g_N)k_{t+1}\end{aligned}$$

Lecture Recap

- ▶ Production Function: $Y = F(K, AN)$.
- ▶ Output per effective worker: $y \equiv Y/AN = \frac{F(K, AN)}{AN} = F\left(\frac{K}{AN}, 1\right) = f(k)$.
- ▶ Assume that A and N grow at constant rates g_A and g_N respectively.
- ▶ Then, to keep K/AN constant, K must grow at the same rate as AN .
- ▶ Capital Accumulation Equation: $K_{t+1} - K_t = sY_t - \delta K_t$.
- ▶ Capital per effective worker accumulation equation:

$$\begin{aligned}\frac{K_{t+1}}{A_{t+1}N_{t+1}} - \frac{K_t}{A_tN_t} &= s\frac{Y_t}{A_tN_t} - \delta\frac{K_t}{A_tN_t} \implies \frac{K_{t+1}}{A_{t+1}N_{t+1}} \cdot \frac{A_{t+1}N_{t+1}}{A_tN_t} - k_t = sy_t - \delta k_t \\ &\implies (1 + g_A + g_N)k_{t+1} - (1 + g_A + g_N)k_t = sf(k_t) - (\delta + g_A + g_N)k_t\end{aligned}$$

Lecture Recap

- ▶ Production Function: $Y = F(K, AN)$.
- ▶ Output per effective worker: $y \equiv Y/AN = \frac{F(K, AN)}{AN} = F\left(\frac{K}{AN}, 1\right) = f(k)$.
- ▶ Assume that A and N grow at constant rates g_A and g_N respectively.
- ▶ Then, to keep K/AN constant, K must grow at the same rate as AN .
- ▶ Capital Accumulation Equation: $K_{t+1} - K_t = sY_t - \delta K_t$.
- ▶ Capital per effective worker accumulation equation:

$$\begin{aligned}\frac{K_{t+1}}{A_{t+1}N_{t+1}} - \frac{K_t}{A_tN_t} &= s\frac{Y_t}{A_tN_t} - \delta\frac{K_t}{A_tN_t} \implies \frac{K_{t+1}}{A_{t+1}N_{t+1}} \cdot \frac{A_{t+1}N_{t+1}}{A_tN_t} - k_t = sy_t - \delta k_t \\ &\implies (1 + g_A + g_N)k_{t+1} - (1 + g_A + g_N)k_t = sf(k_t) - (\delta + g_A + g_N)k_t \\ &\implies (1 + g_A + g_N)(k_{t+1} - k_t) = sf(k_t) - (\delta + g_A + g_N)k_t\end{aligned}$$

Lecture Recap

- ▶ Production Function: $Y = F(K, AN)$.
- ▶ Output per effective worker: $y \equiv Y/AN = \frac{F(K, AN)}{AN} = F\left(\frac{K}{AN}, 1\right) = f(k)$.
- ▶ Assume that A and N grow at constant rates g_A and g_N respectively.
- ▶ Then, to keep K/AN constant, K must grow at the same rate as AN .
- ▶ Capital Accumulation Equation: $K_{t+1} - K_t = sY_t - \delta K_t$.
- ▶ Capital per effective worker accumulation equation:

$$\begin{aligned}\frac{K_{t+1}}{A_{t+1}N_{t+1}} - \frac{K_t}{A_tN_t} &= s\frac{Y_t}{A_tN_t} - \delta\frac{K_t}{A_tN_t} \implies \frac{K_{t+1}}{A_{t+1}N_{t+1}} \cdot \frac{A_{t+1}N_{t+1}}{A_tN_t} - k_t = sy_t - \delta k_t \\ &\implies (1 + g_A + g_N)k_{t+1} - (1 + g_A + g_N)k_t = sf(k_t) - (\delta + g_A + g_N)k_t \\ &\implies (1 + g_A + g_N)(k_{t+1} - k_t) = sf(k_t) - (\delta + g_A + g_N)k_t\end{aligned}$$

- ▶ Steady state condition: $sf(k^*) = (\delta + g_A + g_N)k^*$.