Math Recap

(Log-Approximation)

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- Note that $(1 + g_X)(1 + g_Y) \approx 1 + g_X + g_Y$.

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(Solow Model)

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- Steady state condition: $sf(k^*) = (\delta + g_A + g_N)k^*$.