THE UNIVERSITY OF MELBOURNE

Setup

OLG Model with MiU

- Time is discrete and infinite: $t = 0, 1, 2, \ldots$.
- Each agent of generation t lives for two periods: young in period t and old in period $t + 1$.
- As such, at each period t, there are two generations alive: the young generation t and the old generation $t - 1$.
- Each generation is of size N.
- Young agents work and are paid a wage w_t. No consumption when young!
- Agents can save by holding money m_{t+1} or capital k_{t+1}.
- Capital earns a real interest rate r_{t+1} with certainty. Money yields no nominal return.
- Denote price level as P_t and money supply as M_t.

Individuals

OLG Model with MiU

- Aims to maximise lifetime utility:

$$u_t = \theta \log\left(\frac{m_{t+1}}{P_t}\right) + (1-\theta)\log\left(c_{ot+1}\right), \quad \theta \in [0,1]$$

- Budget constraint when young:

$$k_{t+1} + \frac{m_{t+1}}{P_t} = w_t$$

- Budget constraint when old:

$$c_{ot+1} = (1+r_{t+1})k_{t+1} + \frac{m_{t+1}}{P_{t+1}} = (1+r_{t+1})k_{t+1} + \frac{m_{t+1}}{P_t}\cdot\frac{P_t}{P_{t+1}}$$

- Substituting using k_{t+1}, we can write:

$$c_{ot+1} = (1+r_{t+1})\left(w_t - \frac{m_{t+1}}{P_t}\right) + \frac{m_{t+1}}{P_t}\cdot\frac{P_t}{P_{t+1}} = (1+r_{t+1})w_t - \left(1 + r_{t+1} - \frac{P_t}{P_{t+1}}\right)\cdot\frac{m_{t+1}}{P_t}$$

Individuals

OLG Model with Money

- Lifetime budget constraint:

$$c_{o,t+1} = (1 + r_{t+1})w_t - \left(1 + r_{t+1} - \frac{P_t}{P_{t+1}}\right)\frac{m_{t+1}}{P_t}.$$

- Let $x \equiv \dfrac{m_{t+1}}{P_t}$ (real money balances when young).
- The individual's maximisation problem becomes:

$$\max_x \ \theta \log\,(x) + (1-\theta)\log\left[(1 + r_{t+1})w_t - \left(1 + r_{t+1} - \frac{P_t}{P_{t+1}}\right)x\right].$$

- Taking the first-order condition w.r.t. x, we get:

$$\theta \cdot \frac{1}{x} + (1-\theta) \cdot \frac{1}{(1 + r_{t+1})w_t - \left(1 + r_{t+1} - \frac{P_t}{P_{t+1}}\right)x} \cdot \left(-\left(1 + r_{t+1} - \frac{P_t}{P_{t+1}}\right)\right) = 0.$$

Individuals

OLG Model with Money

- From last slide:

$$\theta \cdot \frac{1}{x} + (1-\theta) \cdot \frac{1}{(1+r_{t+1})w_t - \left(1 + r_{t+1} - \frac{P_t}{P_{t+1}}\right)x} \cdot \left(-\left(1 + r_{t+1} - \frac{P_t}{P_{t+1}}\right)\right) = 0$$

$$\implies \frac{\theta}{x} = \frac{(1-\theta)\left(1 + r_{t+1} - \frac{P_t}{P_{t+1}}\right)}{(1+r_{t+1})w_t - \left(1 + r_{t+1} - \frac{P_t}{P_{t+1}}\right)x}$$

$$\implies \theta \cdot \left[(1+r_{t+1})w_t - \left(1 + r_{t+1} - \frac{P_t}{P_{t+1}}\right)x\right] = (1-\theta)\left(1 + r_{t+1} - \frac{P_t}{P_{t+1}}\right)x$$

$$\implies \theta(1+r_{t+1})w_t - \theta\left(1 + r_{t+1} - \frac{P_t}{P_{t+1}}\right)x = (1-\theta)\left(1 + r_{t+1} - \frac{P_t}{P_{t+1}}\right)x$$

$$\implies \theta(1+r_{t+1})w_t = \left(1 + r_{t+1} - \frac{P_t}{P_{t+1}}\right)x$$

$$\implies x = \frac{\theta(1+r_{t+1})w_t}{1 + r_{t+1} - \frac{P_t}{P_{t+1}}}$$

OLG Model with Money

Fisher equation and money demand

- Recall the Fisher equation:

$$1 + i_{t+1} = (1 + r_{t+1})\frac{P_{t+1}}{P_t} \Leftrightarrow 1 + r_{t+1} = (1 + i_{t+1}) \cdot \frac{P_t}{P_{t+1}}$$

- Using this, we can rewrite money demand as:

$$x = \frac{\theta(1 + r_{t+1})w_t}{1 + r_{t+1} - \frac{P_t}{P_{t+1}}} \Leftrightarrow \frac{m_{t+1}}{P_t} = \frac{\theta \cdot (1 + i_{t+1}) \cdot \frac{P_t}{P_{t+1}} \cdot w_t}{(1 + i_{t+1}) \cdot \frac{P_t}{P_{t+1}} - \frac{P_t}{P_{t+1}}}$$

$$\Longrightarrow \frac{m_{t+1}}{P_t} = \frac{\theta \cdot (1 + i_{t+1}) \cdot \frac{P_t}{P_{t+1}} \cdot w_t}{i_{t+1} \cdot \frac{P_t}{P_{t+1}}}$$

$$\Longrightarrow \frac{m_{t+1}}{P_t} = \theta \left(1 + \frac{1}{i_{t+1}}\right) w_t$$