Content Review

(Leverage Cycles)

Model Primitives

- **Agents:** are indexed by h and equally distributed over $h \in [0, 1]$.
- **Asset:** a single type of asset Y.
- **Timing:** 2 periods. In period 1, agents decide whether to buy or sell asset Y.
- **Future States:** 2 types $\{U, D\}$.
- **Beliefs:** Individual h puts probability h on state U and $1 - h$ on state D.
- **Endowment:** individuals are endowed with one unit of money and one unit of asset Y.

Model Primitives

- **Risk Profile:** individuals are risk-neutral and want to maximize their wealth.
- **Asset payoffs:** a unit of asset Y pays 1 in state U and 0.2 in state D.
- **Strategy:** individuals buy if they believe an asset is underpriced (undervalued).

$$\begin{cases} \text{Buy} & \text{if } p < h \cdot 1 + (1-h) \cdot 0.2 \\ \text{Sell} & \text{if } p > h \cdot 1 + (1-h) \cdot 0.2 \\ \text{Indifferent} & \text{if } p = h \cdot 1 + (1-h) \cdot 0.2 \end{cases}$$

- No short selling!

No-Borrowing Equilibrium

- For any price, there is an indifferent individual h.
- The equilibrium price makes h^* indifferent between buying and selling, i.e.

$$h^* \cdot 1 + (1 - h^*) \cdot 0.2 = 0.8h^* + 0.2 = p^* \Leftrightarrow h^* = \frac{p^* - 0.2}{0.8} \tag{1}$$

- Individuals $h > h^*$ buy. Each buy $= \frac{\text{Total Money}}{\textit{Price}} = \frac{1}{p^*}$. Demand $= \frac{1}{p^*}(1 - h^*)$
- Individuals $h < h^*$ sell the asset. Supply $= h^*$.
- In equilibrium, demand $=$ supply.

$$\frac{1}{p^*}(1 - h^*) = h^* \tag{2}$$

No-Borrowing Equilibrium

We solve (1) and (2) to pin down p^* and h^*.

From equation (2), we have

$$\frac{1}{p^*}(1-h^*) = h^* \Leftrightarrow 1-h^* = p^* h^* \Leftrightarrow h^* = \frac{1}{1+p^*}$$

Substituting h^* into equation (1), we have

$$\frac{1}{1+p^*} = \frac{p^*-0.2}{0.8} \Leftrightarrow 0.8 = (1+p^*)(p^*-0.2) \Leftrightarrow 0.8 = p^{*2} + 0.8p^* - 0.2$$

This gives a quadratic expression in terms of p^*

$$p^{*2} + 0.8p^* - 1 = 0 \implies p^* = \frac{-2 \pm \sqrt{29}}{5} = 0.677, -1.477 \implies p^* = 0.68.$$

$$\implies h^* = \frac{1}{1+0.677} = 0.60.$$

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Borrowing Equilibrium

- If unrestricted borrowing, $h = 1$ would borrow to buy the asset pushing $p^* = 1$.
- Recall $p^* = 0.8h^* + 0.2$. Thus, $p^* \in [0.2, 1]$.
- Under restricted borrowing, assume individuals can borrow 0.2 for each unit of collateral.
- Demand is now different

$$\frac{1}{p^*}\left(\underbrace{(1-h)\cdot 1}_{endowment} + \underbrace{0.2\cdot 1}_{borrowing}\right)$$

- Still, $h < h^*$ individuals willing to sell. So, supply $= h^*$.
- In equilibrium, demand $=$ supply.

$$h^* = \frac{1}{p^*}(1 - h^* + 0.2) \tag{3}$$

Borrowing Equilibrium

Here, we solve (1) and (3) to pin down p^* and h^*.

From equation (3), we have

$$h^* = \frac{1}{p^*}(1 - h^* + 0.2) \Leftrightarrow p^* h^* = 1 - h^* + 0.2 \Leftrightarrow h^* = \frac{1.2}{1 + p^*}$$

Substituting h^* into equation (1), we have

$$\frac{1.2}{1 + p^*} = \frac{p^* - 0.2}{0.8} \Leftrightarrow 0.96 = (1 + p^*)(p^* - 0.2) \Leftrightarrow 0.96 = p^{*2} + 0.8p^* - 0.2$$

This gives a quadratic expression in terms of p^*

$$p^{*2} + 0.8p^* - 1.16 = 0 \implies p^* = \frac{-2 \pm \sqrt{33}}{5} = 0.75, -1.55 \implies p^* = 0.75.$$

$$\implies h^* = \frac{1.2}{1 + 0.75} = 0.69.$$

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Takeaways

- Borrowing allows most optimistic individuals to own assets – this raises prices.
- Loosens the borrowing constraints, $\uparrow h^*, \uparrow p^*$.
- Thus, asset prices $\neq$ fundamental value; rather dependent on borrowing constraints (leverage).
- Leverage cycles mostly responsible for asset price fluctuations.
- Regulation preventing big leverage cycles can prevent asset price cycles.