Content Review

(Diamond-Dybvig Model)

Diamond-Dybvig Model

Assumptions

- Time begins at 0. Two-period model.
  - Period 0. Funds transferred from savers to investors. Investment takes place.
  - Period 1. Liquidity shock is realized. Impatient savers want to consume immediately.
  - Period 2. Investment comes to fruition. Profits are distributed.
- Savers have 1 unit of funds. Savers realize whether they are *patient* or *impatient* after liquidity shock.
  - Utility depends on saver type

$$U(c_1, c_2) = \begin{cases} u(c_1) & \text{if impatient} \\ \beta u(c_1 + c_2) & \text{if patient} \end{cases}$$

  - Utility is strictly concave $\Leftrightarrow$ strictly increasing $u'(c) > 0$ but at a decreasing rate $u''(c) < 0$.

Diamond-Dybvig Model

Assumptions

- $\beta < 1 \implies$ agents prefer early consumption.
- $R\beta > 1 \implies$ but patient individuals are willing to invest.
  - PDV of investment > PDV of immediate consumption.
  - $R\beta > 1 \implies \beta u(R) > u(\beta R) > u(1)$ since utility is strictly concave.
- θ is the probability that a saver is impatient.
- Key Mechanism:
  - Project requires 1 unit of money to start, two periods to complete.
  - If completed, payoff is R. Option to cancel project in period 1 and return investment.
  - Investments less liquid than savings, creates a liquidity mismatch.

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Diamond-Dybvig Model

No Banks

- Period 0. Everyone invests in a project.
- Period 1. Liquidity shock realized. θ fraction of agents are impatient, cancel project, get their investment back and consume $c_1 = 1$.
- Period 2. Patient agents get payoff and consume $c_2 = R$.
- Expected utility of agents $= \theta \cdot u(1) + (1 - \theta) \cdot \beta u(R)$.

Diamond-Dybvig Model

Social Planner Problem

Social planner wants to maximize expected utility of agents given a budget constraint.

$$\max_{c_1, c_2} \quad \theta \cdot u(c_1) + (1-\theta) \cdot \beta u(c_2) \quad \text{s.t.} \quad (1-\theta)c_2 \leq (1-\theta c_1) \cdot R$$

$$\implies L = \theta \cdot u(c_1) + (1-\theta) \cdot \beta u(c_2) + \lambda((1-\theta c_1) \cdot R - (1-\theta)c_2)$$

$$\implies \frac{\partial L}{\partial c_1} = \theta u'(c_1) - \lambda \theta R = 0 \Leftrightarrow \lambda = \frac{u'(c_1)}{R}$$

$$\implies \frac{\partial L}{\partial c_2} = \beta(1-\theta)u'(c_2) - \lambda(1-\theta) = 0 \Leftrightarrow \beta u'(c_2) = \lambda \Leftrightarrow \beta R u'(c_2) = u'(c_1).$$

Since $\beta R > 1 \implies u'(c_1) > u'(c_2) \Leftrightarrow c_1^* < c_2^*$.

Moreover, $1 < c_1^* < c_2^* < R$ because $u(\cdot)$ is strictly concave.