Kiyotaki-Wright Model

Assumptions

- Unit measure of agents $[0, 1]$. Infinite (or very large) population.
- Unit measure of goods produced $[0, 1]$. Only one good.
- Discrete time $t = 1, 2, 3, ...$, infinitely lived agents.
- Agents do not consume the good the produce. They trade it.

Kiyotaki-Wright Model

Assumptions

- Each period, matches occur between agents. Trade happens if mutually beneficial.
- Probability an individual is interested in trading is x. So the probability of a trade is x^2.
- Successful trade gives a utility of u. Cost of production is k.
- Assume $u > k$ so that production occurs.

Kiyotaki-Wright Model

Without Money

Expected utility each period is given by:

$$
\begin{aligned}
V &= x^2 \cdot (u-k) + (1-x^2) \cdot 0 + \beta \cdot V \\
\implies V &= x^2 \cdot (u-k) + \beta \cdot V \\
\implies (1-\beta)V &= x^2(u-k) \\
\implies V &= \frac{x^2(u-k)}{1-\beta}
\end{aligned}
$$

Kiyotaki-Wright Model

With Money

- A proportion M is endowed with money. $0 < M < 1$.
- Agents can hold either a unit of money or goods.
- Agents choose to accept money with probability π and believe that a random agent will accept money with probability Π.
- Agents holding goods get value V_C and those holding money get value V_M.

Kiyotaki-Wright Model

With Money

What is the optimal level of π for an agent? $\implies$ compare V_C and V_M.

$$
\begin{aligned}
V_C &= (1-M)\cdot x^2\cdot(u-k) + M\cdot x\cdot\pi\cdot\beta V_M + (1-M\cdot x\cdot\pi)\cdot\beta V_C \\
V_M &= (1-M)\cdot x\cdot\Pi\cdot(u-k) + (1-M)\cdot x\cdot\Pi\cdot\beta V_C + \{1-(1-M)\cdot x\cdot\Pi\}\cdot\beta V_M
\end{aligned}
$$

which leads to solution of the form:

$$
\begin{aligned}
&\textit{if} \quad x < \Pi \quad \implies V_C < V_M \quad \implies \pi = 1 \\
&\textit{if} \quad x > \Pi \quad \implies V_C > V_M \quad \implies \pi = 0 \\
&\textit{if} \quad x = \Pi \quad \implies V_C = V_M \quad \implies \pi \in [0,1]
\end{aligned}
$$