Cagan's Model of Hyperinflation

Exchange Equation: $M_t \cdot V_t = P_t \cdot Y_t \Leftrightarrow \frac{M_t}{P_t} = \frac{Y_t}{V_t}$

Velocity - Interest Rate: $\log V_t = \alpha i_t$

Fisher Equation: $i_t = r_t + \pi_t^e$

By definition: $\pi_t = p_t - p_{t-1}$; $\pi_t^e = p_t^e - p_{t-1}^e$ where $p_t = \log P_t$

Thus, we can write the exchange equation as:

$$\frac{M_t}{P_t} = \frac{Y_t}{V_t} \underset{\text{taking logs}}{\Longrightarrow} \log M_t - \log P_t = \log Y_t - \log V_t$$

$$\Longrightarrow m_t - p_t = y_t - \alpha i_t$$

$$\Longrightarrow m_t - p_t = y_t - \alpha r_t - \alpha \pi_t^e$$

$$\Longrightarrow m_t - p_t = \gamma - \alpha \pi_t^e$$

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Thus, we have changes in real money demand as a function of inflation expectations.

$$m_t - p_t = \gamma - \alpha_2 \pi_t^e \Leftrightarrow \pi_t^e = \frac{1}{\alpha_2}(\gamma - m_t + p_t) \qquad \text{where } \alpha_2 > 0$$

- Demand for money is negatively linked with inflation as cost of holding money increases.
- Since inflation expectations is not observable, the above equation cannot be directly estimated.
- A proxy is used to calculate expected inflation

$$\pi_t^e = \lambda \pi_{t-1} + (1-\lambda)\pi_{t-1}^e$$

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Recall that:

$$\begin{aligned} & m_t - p_t = \gamma - \alpha_2 \pi_t^e \\ \Longrightarrow\ & m_t - p_t = \gamma - \alpha_2(\lambda \pi_{t-1} + (1-\lambda)\pi_{t-1}^e) \end{aligned}$$

Recall that $\pi_{t-1}^e = \frac{\gamma - (m_{t-1} - p_{t-1})}{\alpha_2}$

$$\begin{aligned} \Longrightarrow\ & m_t - p_t = \gamma - \alpha_2 \left(\lambda \pi_{t-1} + (1-\lambda) \frac{\gamma - (m_{t-1} - p_{t-1})}{\alpha_2} \right) \\ \Longrightarrow\ & m_t - p_t = \lambda(\gamma - \alpha_2 \pi_{t-1}) + (1-\lambda)(m_{t-1} - p_{t-1}) \end{aligned}$$

- The variables on the right hand side are observed and can be estimated.

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Note that we can rewrite that equation in terms of p_t as:

$$
\begin{aligned}
p_t &= \frac{-\lambda\gamma + m_t + (-\alpha_2\lambda + 1 - \lambda)p_{t-1} - (1-\lambda)m_{t-1}}{1-\alpha_2\lambda} \\
&= -\frac{\lambda\gamma - m_t + (1-\lambda)m_{t-1}}{1-\alpha_2\lambda} + \frac{-\alpha_2\lambda + 1 - \lambda}{1-\alpha_2\lambda}p_{t-1} \\
&= A + B \cdot p_{t-1}
\end{aligned}
$$

* Stability of the system depends on the value of B!

* That is, if $|B| < 1$, the price level converges to a steady state.