THE UNIVERSITY OF MELBOURNE

Continuous Probability Distributions

1. Chi-square Distribution: if $Z_1, Z_2, \ldots, Z_k$ are independent standard normal random variables, then $V = \sum_{i=1}^{k} Z_i^2 \sim \chi_k^2$ with k degrees of freedom.
2. t-distribution: if $Z \sim N(0, 1)$, $V \sim \chi_k^2$ where Z and V are independent, then $t = \frac{Z}{\sqrt{V/k}} \sim t_k$ with k degrees of freedom.
3. F-distribution: if V_1 and V_2 are independent chi-square random variables with k_1 and k_2 degrees of freedom, then $F = \frac{V_1/k_1}{V_2/k_2} \sim F_{k_1,k_2}$ with k_1 and k_2 degrees of freedom.

Testing for Population Variance

- Chi-square test for population variance σ^2.

$$\frac{(n-1)s^2}{\sigma^2} \sim \chi^2_{n-1} \Leftrightarrow \text{ CI: } \left(\frac{(n-1)s^2}{\chi^2_{\alpha/2,n-1}}, \frac{(n-1)s^2}{\chi^2_{1-\alpha/2,n-1}} \right) \Leftrightarrow \chi^2_{obs} = \frac{(n-1)s^2}{\sigma_0^2}$$

- F-test for ratio of two population variances σ_1^2/σ_2^2.

$$\frac{s_1^2/\sigma_1^2}{s_2^2/\sigma_2^2} \sim F_{n_1-1,n_2-1} \Leftrightarrow \text{ CI: } \left(\frac{s_1^2/s_2^2}{F_{\alpha/2,n_1-1,n_2-1}}, \frac{s_1^2/s_2^2}{F_{1-\alpha/2,n_1-1,n_2-1}} \right) \Leftrightarrow F_{obs} = \frac{s_1^2}{s_2^2}$$

THE UNIVERSITY OF MELBOURNE

Testing for Population Proportions

- One-sample proportion test:

$$\hat{p} \sim \mathcal{N}\left(\mu_{\hat{p}}, \sigma_{\hat{p}}\right) \Leftrightarrow \text{ CI: } \hat{p} \pm z_{\alpha/2} s_{\hat{p}} \text{ where } s_{\hat{p}} = \sqrt{\frac{\hat{p}\hat{q}}{n}} \Leftrightarrow Z_{obs} = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0 q_0}{n}}} \sim \mathcal{N}(0,1)$$

- Two-sample proportion test:

$$\hat{p}_1 - \hat{p}_2 \sim \mathcal{N}\left(\mu_{\hat{p}_1 - \hat{p}_2}, \sigma_{\hat{p}_1 - \hat{p}_2}\right) \Leftrightarrow \text{ CI: } (\hat{p}_1 - \hat{p}_2) \pm z_{\alpha/2} s_{\hat{p}_1 - \hat{p}_2} \Leftrightarrow Z_{obs} = \frac{(\hat{p}_1 - \hat{p}_2) - D_0}{s_{\hat{p}_1 - \hat{p}_2}} \sim \mathcal{N}(0,1)$$

$$\text{If } D_0 = 0, \text{ then } s_{\hat{p}_1 - \hat{p}_2} = \sqrt{\hat{p}\hat{q}\left(\frac{1}{n_1} + \frac{1}{n_2}\right)} \text{ where } \hat{p} = \frac{f_1 + f_2}{n_1 + n_2}$$

$$\text{If } D_0 \neq 0, \text{ then } s_{\hat{p}_1 - \hat{p}_2} = \sqrt{\frac{\hat{p}_1 \hat{q}_1}{n_1} + \frac{\hat{p}_2 \hat{q}_2}{n_2}}$$