Content Review

Lecture Review: Leverage Cycles

Model Primitives

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- **Endowment:** individuals are endowed with one unit of money and one unit of asset Y.

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$$\begin{cases} \text{Buy} & \text{if } p < h \cdot 1 + (1-h) \cdot 0.2 \\ \text{Sell} & \text{if } p > h \cdot 1 + (1-h) \cdot 0.2 \\ \text{Indifferent} & \text{if } p = h \cdot 1 + (1-h) \cdot 0.2 \end{cases}$$

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- No short selling!

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No-Borrowing Equilibrium

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- In equilibrium, demand $=$ supply.

$$\frac{1}{p^*}(1 - h^*) = h^* \tag{2}$$

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$$p^{*2} + 0.8p^* - 1 = 0 \implies p^* = \frac{-2 \pm \sqrt{29}}{5} = 0.677, -1.477$$

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$$\implies h^* = \frac{1}{1 + 0.677} = 0.60.$$

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$$h^* = \frac{1}{p^*}(1 - h^* + 0.2) \tag{3}$$

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$$\implies h^* = \frac{1.2}{1 + 0.75} = 0.69.$$

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- Thus, asset prices $\neq$ fundamental value; rather dependent on borrowing constraints (leverage).
- Leverage cycles mostly responsible for asset price fluctuations.
- Regulation preventing big leverage cycles can prevent asset price cycles.