

# Content Review



# Diamond-Dybvig Model

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- ▶ Utility is strictly concave  $\Leftrightarrow$  strictly increasing  $u'(c) > 0$  but at a decreasing rate  $u''(c) < 0$ .



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  - ▶ Investments less liquid than savings, creates a liquidity mismatch.



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- ▶ Period 2. Patient agents get payoff and consume  $c_2 = R$ .
- ▶ Expected utility of agents  $= \theta \cdot u(1) + (1 - \theta) \cdot \beta u(R)$ .



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Social planner wants to maximize expected utility of agents given a budget constraint.





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$$\max_{c_1, c_2} \quad \theta \cdot u(c_1) + (1 - \theta) \cdot \beta u(c_2) \quad \text{s.t.} \quad (1 - \theta)c_2 \leq (1 - \theta c_1) \cdot R$$



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$$\implies \frac{\partial L}{\partial c_1} = \theta u'(c_1)$$



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Since  $\beta R > 1 \implies u'(c_1) > u'(c_2)$



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Since  $\beta R > 1 \implies u'(c_1) > u'(c_2) \Leftrightarrow c_1^* < c_2^*$ .

Moreover,  $1 < c_1^* < c_2^* < R$  because  $u(\cdot)$  is strictly concave.