Content Review

Kiyotaki-Wright Model

Assumptions

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- Unit measure of goods produced $[0, 1]$. Only one good.
- Discrete time $t = 1, 2, 3, ...$, infinitely lived agents.

Kiyotaki-Wright Model

Assumptions

- Unit measure of agents $[0, 1]$. Infinite (or very large) population.
- Unit measure of goods produced $[0, 1]$. Only one good.
- Discrete time $t = 1, 2, 3, ...,$ infinitely lived agents.
- Agents do not consume the good the produce. They trade it.

Kiyotaki-Wright Model

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- Each period, matches occur between agents. Trade happens if mutually beneficial.

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- Successful trade gives a utility of u. Cost of production is k.

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Assumptions

- Each period, matches occur between agents. Trade happens if mutually beneficial.
- Probability an individual is interested in trading is x. So the probability of a trade is x^2.
- Successful trade gives a utility of u. Cost of production is k.
- Assume $u > k$ so that production occurs.

Kiyotaki-Wright Model
Without Money

Expected utility each period is given by:

$$V = x^2 \cdot (u - k) +$$

Kiyotaki-Wright Model

Without Money

Expected utility each period is given by:

$$V = x^2 \cdot (u - k) + (1 - x^2) \cdot 0 +$$

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Without Money

Expected utility each period is given by:

$$V = x^2 \cdot (u - k) + (1 - x^2) \cdot 0 + \beta \cdot V$$

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$$\implies V = x^2 \cdot (u - k) + \beta \cdot V$$

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Without Money

Expected utility each period is given by:

$$
\begin{aligned}
& V = x^2 \cdot (u - k) + (1 - x^2) \cdot 0 + \beta \cdot V \\
\implies & V = x^2 \cdot (u - k) + \beta \cdot V \\
\implies & (1 - \beta) V = x^2 (u - k)
\end{aligned}
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Without Money

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\implies & V = x^2 \cdot (u - k) + \beta \cdot V \\
\implies & (1 - \beta) V = x^2 (u - k) \\
\implies & V = \frac{x^2 (u - k)}{1 - \beta}
\end{aligned}
$$

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- A proportion M is endowed with money. $0 < M < 1$.

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With Money

- A proportion M is endowed with money. $0 < M < 1$.
- Agents can hold either a unit of money or goods.
- Agents choose to accept money with probability π and believe that a random agent will accept money with probability Π.
- Agents holding goods get value V_C and those holding money get value V_M.

Kiyotaki-Wright Model

With Money

What is the optimal level of π for an agent?

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With Money

What is the optimal level of π for an agent? $\implies$ compare V_C and V_M.

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V_C

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What is the optimal level of π for an agent? $\implies$ compare V_C and V_M.

$$V_C = (1 - M) \cdot x^2 \cdot (u - k) +$$

Kiyotaki-Wright Model

With Money

What is the optimal level of π for an agent? $\implies$ compare V_C and V_M.

$$V_C = (1 - M) \cdot x^2 \cdot (u - k) + M \cdot x \cdot \pi \cdot \beta V_M +$$

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What is the optimal level of π for an agent? $\implies$ compare V_C and V_M.

$$V_C = (1 - M) \cdot x^2 \cdot (u - k) + M \cdot x \cdot \pi \cdot \beta V_M + (1 - M \cdot x \cdot \pi) \cdot \beta V_C$$

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$$V_M$$

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$$
\begin{aligned}
V_C &= (1-M)\cdot x^2 \cdot (u-k) + M \cdot x \cdot \pi \cdot \beta V_M + (1 - M \cdot x \cdot \pi)\cdot \beta V_C \\
V_M &= (1-M)\cdot x \cdot \Pi \cdot (u-k) +
\end{aligned}
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$$V_M = (1 - M) \cdot x \cdot \Pi \cdot (u - k) + (1 - M) \cdot x \cdot \Pi \cdot \beta V_C + \{1 - (1 - M) \cdot x \cdot \Pi\} \cdot \beta V_M$$

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which leads to solution of the form:

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which leads to solution of the form:

$$\textit{if} \quad x < \Pi \quad \implies V_C < V_M \quad \implies$$

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&\textit{if} \quad x = \Pi && \implies V_C = V_M && \implies
\end{aligned}$$

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What is the optimal level of π for an agent? $\implies$ compare V_C and V_M.

$$V_C = (1 - M) \cdot x^2 \cdot (u - k) + M \cdot x \cdot \pi \cdot \beta V_M + (1 - M \cdot x \cdot \pi) \cdot \beta V_C$$
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which leads to solution of the form:

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