Content Review

Cagan's Model of Hyperinflation

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$$\Longrightarrow m_t - p_t = \gamma - \alpha_2 \pi_t^e$$

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- Demand for money is negatively linked with inflation as cost of holding money increases.

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- Demand for money is negatively linked with inflation as cost of holding money increases.
- Since inflation expectations is not observable, the above equation cannot be directly estimated.
- A proxy is used to calculate expected inflation

$$\pi_t^e = \lambda \pi_{t-1} + (1 - \lambda)\pi_{t-1}^e$$

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$$m_t - p_t = \gamma - \alpha_2 \pi_t^e$$
$$\implies m_t - p_t = \gamma - \alpha_2(\lambda \pi_{t-1} + (1-\lambda)\pi_{t-1}^e)$$

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Recall that $\pi_{t-1}^e = \frac{\gamma - (m_{t-1} - p_{t-1})}{\alpha_2}$

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\begin{aligned}
m_t - p_t &= \gamma - \alpha_2 \pi_t^e \\
\Longrightarrow m_t - p_t &= \gamma - \alpha_2(\lambda \pi_{t-1} + (1-\lambda)\pi_{t-1}^e)
\end{aligned}
$$

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\begin{aligned}
\Longrightarrow m_t - p_t &= \gamma - \alpha_2 \left(\lambda \pi_{t-1} + (1-\lambda) \frac{\gamma - (m_{t-1} - p_{t-1})}{\alpha_2} \right) \\
\Longrightarrow m_t - p_t &= \lambda(\gamma - \alpha_2 \pi_{t-1}) + (1-\lambda)(m_{t-1} - p_{t-1})
\end{aligned}
$$

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$$\implies m_t - p_t = \gamma - \alpha_2 \left(\lambda \pi_{t-1} + (1-\lambda) \frac{\gamma - (m_{t-1} - p_{t-1})}{\alpha_2} \right)$$

$$\implies m_t - p_t = \lambda(\gamma - \alpha_2 \pi_{t-1}) + (1-\lambda)(m_{t-1} - p_{t-1})$$

- The variables on the right hand side are observed and can be estimated.

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$$
\begin{aligned}
p_t &= \frac{-\lambda\gamma + m_t + (\alpha_2\lambda + 1 - \lambda)p_{t-1} - (1-\lambda)m_{t-1}}{1 - \alpha_2\lambda} \\
&= -\frac{\lambda\gamma - m_t + (1-\lambda)m_{t-1}}{1 - \alpha_2\lambda} + \frac{\alpha_2\lambda + 1 - \lambda}{1 - \alpha_2\lambda}p_{t-1}
\end{aligned}
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&= A + B \cdot p_{t-1}
\end{aligned}
$$