Lecture Review: Leverage Cycles

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- **Endowment:** individuals are endowed with one unit of money and one unit of asset Y.

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- No short selling!

Lecture Review: Leverage Cycles

No-Borrowing Equilibrium

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- In equilibrium, demand $=$ supply.

$$\frac{1}{p^*}(1 - h^*) = h^* \tag{2}$$

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$$p^{*2} + 0.8p^* - 1 = 0 \implies p^* = \frac{-2 \pm \sqrt{29}}{5} = 0.677, -1.477$$

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$$\implies h^* = \frac{1}{1+0.677} = 0.60.$$

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$$h^* = \frac{1}{p^*}(1 - h^* + 0.2) \tag{3}$$

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$$\implies h^* = \frac{1.2}{1 + 0.75} = 0.69.$$

Lecture Review: Leverage Cycles

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- Leverage cycles mostly responsible for asset price fluctuations.
- Regulation preventing big leverage cycles can prevent asset price cycles.