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  - Utility depends on saver type

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  - Utility is strictly concave $\Leftrightarrow$ strictly increasing $u'(c) > 0$ but at a decreasing rate $u''(c) < 0$.

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- Key Mechanism:
  - Project requires 1 unit of money to start, two periods to complete.
  - If completed, payoff is R. Option to cancel project in period 1 and return investment.
  - Investments less liquid than savings, creates a liquidity mismatch.

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Social Planner Problem

Social planner wants to maximize expected utility of agents given a budget constraint.

$$\max_{c_1, c_2} \quad \theta \cdot u(c_1) + (1-\theta) \cdot \beta u(c_2) \quad \text{s.t.} \quad (1-\theta)c_2 \leq (1-\theta c_1) \cdot R$$

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Since $\beta R > 1 \implies u'(c_1) > u'(c_2) \Leftrightarrow c_1^* < c_2^*$.

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Moreover, $1 < c_1^* < c_2^* < R$ because $u(\cdot)$ is strictly concave.