



Cagan's Model of Hyperinflation

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$$\frac{M_t}{P_t} = \frac{Y_t}{\alpha i_t} = \frac{Y_t}{\alpha(r_t + \pi_t^e)} \xRightarrow{\text{taking logs}} \log M_t - \log P_t = \log Y_t - \log r_t - \log \pi_t^e$$

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Thus, we can write the exchange equation as:

$$\begin{aligned} \frac{M_t}{P_t} = \frac{Y_t}{\alpha i_t} = \frac{Y_t}{\alpha(r_t + \pi_t^e)} &\implies \log M_t - \log P_t = \log Y_t - \log r_t - \log \pi_t^e \\ &\implies m_t - p_t = \gamma - \alpha_2 \pi_t^e \end{aligned}$$



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- ▶ Demand for money is negatively linked with inflation as cost of holding money increases.
- ▶ Since inflation expectations is not observable, the above equation cannot be directly estimated.
- ▶ A proxy is used to calculate expected inflation

$$\pi_t^e = \lambda \pi_{t-1} + (1 - \lambda) \pi_{t-1}^e$$



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Recall that $\pi_{t-1}^e = \frac{\gamma - (m_{t-1} - p_{t-1})}{\alpha_2}$

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- The variables on the right hand side are observed and can be estimated.

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$$\begin{aligned} p_t &= \frac{-\lambda\gamma + m_t + (\alpha_2\lambda + 1 - \lambda)p_{t-1} - (1 - \lambda)m_{t-1}}{1 - \alpha_2\lambda} \\ &= -\frac{\lambda\gamma - m_t + (1 - \lambda)m_{t-1}}{1 - \alpha_2\lambda} + \frac{\alpha_2\lambda + 1 - \lambda}{1 - \alpha_2\lambda} p_{t-1} \end{aligned}$$

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