Discussion 9 - Solutions

Topics

- Input Market
- Pareto Efficiency

Input Market Exercises

Exercise 1

Consider the following information for a T-shirt manufacturing firm that can sell as many T-shirts as it wants for $3 per shirt.

Number of Workers	Quantity of Shirts	MPL	TR	MRPL
0	0			
1	30			
2	80			
3	110			
4	135			
5		20		
6	170			
7				30
8				15

1. Fill in all the blanks in the table.
2. Verify that MRPL for this firm can be calculated in two ways: (1) change in the TR from adding another worker and (2) MPL times the price of the output.
3. If this firm must pay a wage rate of $45 per worker per day, how many workers should be hired now? Why?
4. Suppose the wage rate rises to $60 per worker. How many workers should be hired now?
5. Suppose the firm adopts a new technology that doubles output at each level of employment and the price of shirts remains at $3. What is the effect of this new technology on MPL and MRPL? At a wage of $50, how many workers should the firm hire now?

Solution:

Number of Workers	Quantity of Shirts	MPL	TR	MRPL
0	0	-	-	-
1	30	30	90	90
2	80	50	240	150
3	110	30	330	90
4	135	25	405	75
5	155	20	465	60
6	170	15	510	45
7	180	10	540	30
8	185	5	555	15

Firms will hire more labor when the marginal revenue product of labor is greater than the wage rate, and stop hiring as soon as the two values are equal. When the wage is \$45, the firm should hire 6 workers. When the wage is \$60, the firm should hire 5 workers. With the new technology, the MPL and therefore MRPL double. Now, the firm should hire 7 workers.

Exercise 2

Willy Wonka's Chocolate factory produces chocolate bars by using labor and boats that float down the chocolate river (capital). Assume that Willy Wonka is a price taker in the markets for chocolate bars, labor and boats.

1. Suppose the price of a chocolate bar is \$5 and at the current level of labor and capital $MP_L = 10$, $MP_K = 100$, the price of labor is \$40 and the rental price of a boat is \$600. Assuming the chocolate factory exhibits diminishing marginal returns to both labor and capital, what should Willy Wonka do to improve his profits?

   Solution: Since $MP_L * P_x = 10 * 5 = 50 > 40$, *Willy Wonka should hire more labor. On the other hand, since* $MP_K * P_x = 100 * 5 = 500 < 600$, *he should rent fewer boats.*

2. Charlie eventually takes over the chocolate factory and ensures that it always maximizes profits. Years later the price of a chocolate bar is \$6 and the wage of a worker is \$72. Calculate the MP_L under Charlie's management.

   Solution: $P_L = MRP_L \implies 72 = 6 * MP_L \implies MP_L = 12$.

Exercise 3

Consider the retail gasoline market, which is perfectly competitive. Market demand and supply for gasoline are represented by the following:

$$Supply : P = 0.2Q$$

$$Demand : P = 400 - 0.2Q$$

P is the price of gasoline and Q is barrels of gasoline. There are 100 identical firms in the market. Each gas station hires workers in a perfectly competitive labor market. The supply and demand for labor are represented by:

$$Supply : W = 0.03L$$

$$Demand : W = 50 - 0.02L$$

W is the price of labor (wage) and L is the quantity of workers.

1. How many workers will be hired by each firm in equilibrium?

   Solution: Equating supply and demand in the labor market we can solve for the equilibrium quantity of labor in the market:

   $$0.03L = 50 - 0.02L \implies L^* = 1000$$

   Each firm will hire 1000/100=10 workers.

2. What is the market equilibrium wage?

   Solution: Using the information from part (a), we see that L = 1000. Plugging this into either the supply or demand of the labor market gets that $W = 0.03 * 1000 = 50 - 0.02 * 1000 = \$30$

3. What is the market equilibrium price of gasoline?

   Solution: Using information from this question, we know market demand and supply of this gasoline market. Then we can solve for its equilibrium price and quantity: $0.2Q = 400 - 0.2Q \Rightarrow 400 = 0.4Q \Rightarrow Q = 1000$. *Plug Q = 1000 into its supply:* $P_x = 0.2Q = 0.2 * 1000 = \$200$

4. Calculate the marginal product of labor for each firm.

   Solution: In equilibrium, we know that $P_L = MRP_L = MP_L * P_x$. *We have* $P_L = W = \$30$. *From part (c), we can find* $P_x = \$200$. *Therefore we can get* $MP_L = MRP_L/P_x = \$30/\$200 = 0.15$

Exercise 4

Suppose Dr. Wells owns a running machine company called STAR. He hires technicians (called Cisco, capital) and normal workers (called Barry, labor) to produce treadmills. The marginal product of Cisco is $MP_K = 30 + 3L$ and marginal product of Barry is $MP_L = 20 - L$. L is the amount of labor.

1. In equilibrium, wage of Cisco is three times as much as wage of Barry. How many Barry workers will be hired?

   Solution: In equilibrium, $MP_K/P_K = MP_L/P_L$. *Thus,* $(30 + 3L)/P_K = (20 - L)/P_L$. *In this question, we also know* $P_K = 3 * PL$. *Then we have* $(30 + 3L)/(3P_L) = (20 - L)/P_L$. *Therefore, L=5.*

2. When the number of Barry workers increases, what happens to MP_K? Increase or decrease? What is the relationship between these two inputs?

   Solution: Since $MP_K = 30 + 3L$, MP_K increases when L increases. It means that these two inputs are complementary. (More Barry increases the productivity of Cisco!)

3. If the wage of Cisco increases, does Dr.Wells hire more Barry workers? Or fewer Barry workers? Why? [Hint: Consider both output effect and factor substitution effect!]

   Solution: Output effect: When wage of Cisco increases ($P_K \uparrow$), cost of producing treadmills increases. This firm will produce fewer and thus hire fewer Barry workers. Factor substitution effect: When wage of Cisco increases ($P_K \uparrow$), this firm hires fewer Cisco. Then this firm will hire fewer its complementary input Barry workers too. The two effects combined: This firm hires fewer Barry workers.

Pareto Efficiency Exercises

Exercise 5

Hurley and Leonard pool their money to buy a lottery ticket and manage to win one million dollars. Which of the following is a Pareto efficient division of the winnings, assuming Hurley and Leonard both want as much money as possible?

1. Hurley and Leonard split the money fifty-fifty.

   Solution: This is Pareto efficient: in order to give Hurley more money we must take some from Leonard, and vice versa.

2. Hurley gets all the money and Leonard gets nothing. *Solution: This is also Pareto efficient since to make Leonard better o we must take money from Hurley (and we cannot make Hurley better off at all)*

3. Leonard gets \$400,000, Hurley gets \$300,000, and the other \$300,000 is burned. *Solution: This is not Pareto efficient: instead of burning the other \$300,000 we could give it to Leonard or Hurley.*

Multiple Choice Exercises - Input Markets

Exercise 6

Suppose the marginal product of labor for a coffee producer is given by the equation $MP_L = 15 - L$, where L is the amount of labor. Let the price of a bag of coffee be \$10. What is the equation for MRP_L?

(a) $MRP_L = 15 - 10L$

(b) $MRP_L = 150 - 10L$

(c) $MRP_L = 150 - L$

(d) $MRP_L = 10$

Solution: The correct answer is (b). Since $MRP_L = MP_L * P_x$, we multiply our marginal product line by the price of the output to get the marginal revenue product.

Exercise 7

Continuing from exercise 6, let the market wage be \$20. How much labor should the firm hire?

(a) 2 workers

(b) 0 workers

(c) 13 workers

(d) 15 workers

Solution: The correct answer is (c). Profit-maximizing firms will hire workers until the marginal revenue product of labor is equal to the market wage. Thus, $w = MRP_L \implies 20 = 150 - 10L \implies L = 13$.

Professor Hansen needs to decide how many research assistants to hire. Each paper produced is worth of \$20. Use the following table to answer the next two questions.

Total number of papers produced	Research assistants hired
10	1
18	2
24	3
28	4
30	5

Exercise 8

What is the marginal revenue product of the 5th research assistant hired?

(a) \$20

(b) \$40

(c) 2

(d) \$600

*Solution: The correct answer is (b). The marginal products of the 1st, 2nd, 3rd, 4th and **5th** research assistants are 10, 8, 6, 4, and **2**, respectively. The marginal revenue products are $MP * P = MP * \$20$, so MRP are \$200, \$160, \$120, \$80, and **\$40** respectively.*

Exercise 9

Continuing from the previous exercise, suppose the salary of research assistant is \$80. How many research assistants should Professor Hansen hire?

(a) 5

(b) 2

(c) 3

(d) 4

Solution: The correct answer is (d). Profit is maximized when MRP = wage, and we can check the computed MRP values in (a) to see that this happens after hiring the 4th research assistant.

Multiple Choice Exercises - Pareto Efficiency

Exercise 10

An economy contains two people and two goods, apples and bananas. Person 1 likes apples and dislikes bananas (the more bananas she has, the worse off she is), and person 2 likes bananas and dislikes apples. There are 100 apples and 100 bananas available. Which of the following allocations is Pareto efficient?

(a) person 1 has all the apples and person 2 has all the bananas

(b) person 1 has all the apples and all the bananas

(c) person 2 has all the apples and all the bananas.

(d) Person 1 has 50 apples and 50 bananas. Person 2 also has 50 apples and 50 bananas.

Solution: The correct answer is (a). The only allocation that is Pareto efficient is that in which person 1 has all the apples and person 2 has all the bananas. For any other allocation, one of the persons has some units of the good she does not like, and would be better off not having them while the other person who likes that good would be better off if they could get those units.

Exercise 11

An economy contains two people and two goods, apples and bananas. Person 1 likes apples and doesn't care about bananas (she is indifferent between any bundles (a,b)

and (a,b'), where a is some number of apples and b and b' are numbers of bananas). Person 2 likes bananas and doesn't care about apples. There are 100 apples and 100 bananas available. Which of the following allocations is Pareto efficient?

(a) person 1 has all the apples and person 2 has all the bananas

(b) person 1 has all the apples and all the bananas

(c) person 2 has all the apples and all the bananas.

(d) Person 1 has 50 apples and 50 bananas. Person 2 also has 50 apples and 50 bananas.

Solution: The correct answer is (a). The only allocation that is Pareto efficient is that in which person 1 has all the apples and person 2 has all the bananas. For any other allocation, one of the persons has some units of the good about which she doesn't care; transferring those units to the other person would have no effect on her and would simultaneously make the other person better off.

Exercise 12

An economy contains two people and two goods, apples and bananas. Both people like both goods, but value them differently. For person 1, 1 apple is exactly equivalent to 2 bananas: she is indifferent between any bundles (a, b) and (a-n, b+2n), where a is some number of apples, b is some number of bananas, and n is some number). For person 2, 2 apples are exactly equivalent to 1 banana.

(a) person 1 has all the apples and person 2 has all the bananas

(b) person 1 has all the apples and all the bananas

(c) person 2 has all the apples and all the bananas.

(d) All of the above

Solution: The correct answer is (d). An allocation is Pareto efficient if and only if: either person 1 has no bananas or person 2 has no apples. Why? Suppose person 1 has some bananas and person 2 has some apples. Then by transferring one banana from person 1 to person 2 and one apple from person 2 to person 1 we make both of them better off. On the other hand, if person 1 has no bananas then any trade that makes her better off must involve her getting at least twice as many bananas as she gives up in apples, which results in person 2 being worse off. Similarly, if person 2 has no apples then any trade that makes her better off must involve her getting at least twice as many apples as she gives up in bananas, which results in person 1 being worse off.