Discussion 5

1 Topics

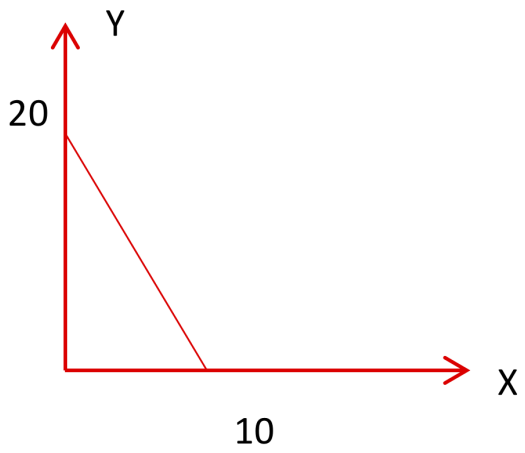
- Budget Constraints
- Utility maximization
- Midterm 1 - Discussion of most missed questions

2 Exercises

2.1 Budget Constraint.

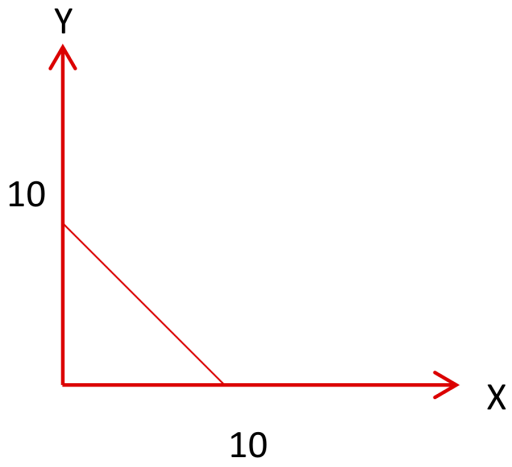
1. Suppose Michelle's income is \$100 per week and she only consumes oranges (good X) and apples (good Y). Each apple costs \$5 and each orange costs \$10.

1. Find the equation for Michelle's budget constraint and graph it.
   Solution:
   $$10X + 5Y = 100$$



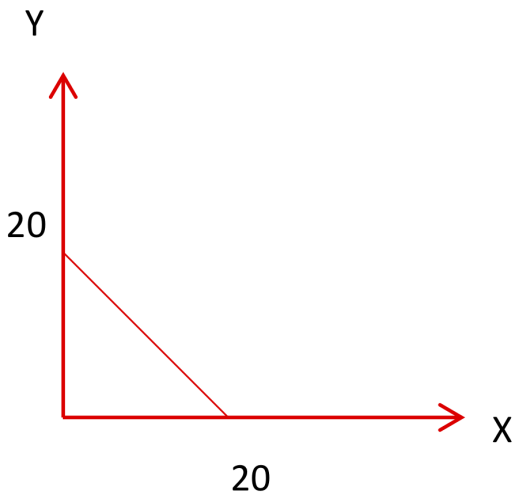
2. Is the bundle $(7, 6)$ affordable for Michelle? What about the bundle $(7, 7)$?
   Solution: Since the bundle $(7, 6)$ *costs* $\$70 + \$30 = \$100$*, it is affordable. However, the bundle* $(7, 7)$ *costs \$105. So it is not affordable.*

3. Now suppose that price of an apple increases from \$5 to \$10. Graph Michelle's new budget constraint and find the equation.
   Solution:
   $$10X + 10Y = 100$$



4. Michelle got a raise and her income is now \$200. Find the equation for Michelle's new budget constraint and graph it.
Solution:

$$10X + 10Y = 200$$



2.2 Utility maximization.

2. Mark spent \$80 to buy 10 cans of beer (good X) and 5 bottles of wine (good Y) last week. Each can of beer cost P_x and each bottle of wine cost P_y. Suppose he is a utility maximizing agent.

1. Now, the price of one bottle of beer has increased by \$1 and he has spent the same amount of money to buy 8 cans of beer and 5 bottles of wine. Find the price of beer before the change and the price of wine.
Solution: We have two budget equations: $10P_x + 5P_y = 80$, $8(P_x + 1) + 5P_y = 80$. *By subtracting both equations, we get* $2P_x = 8$. *So,* $P_x = 4$. *Plugging* $P_x = 4$ *in one of any budget equations gives* $P_y = 8$ *Thus* $P_x = 4$,$P_y = 8$.

2. What is the MRS before the price change and after the price change?
Solution: Before the change:

$$MRS = \frac{MU_X}{MU_Y} = \frac{P_x}{P_y} = \frac{4}{8} = \frac{1}{2}$$

After the change:

$$MRS = \frac{MU_X}{MU_Y} = \frac{P_x + 1}{P_y} = \frac{4+1}{8} = \frac{5}{8}$$

3. Charles derives utility from pairs of black shoes (good X) and pairs of blue jeans (good Y). The marginal utility of a pair of black shoes is $MU_X = \frac{1}{X}$. The marginal utility of a pair of blue jeans is 1. He has an income of \$120. Suppose a pair of black shoes costs \$20, and a pair of blue jeans costs \$40. Which bundle should he consume if he wants to maximize his utility?
Solution: MRS of black shoes for blue jeans is

$$MRS = \frac{MU_X}{MU_Y} = \frac{1}{X}$$

The price ratio between the two goods (or the slope of the budget constraint) is

$$\frac{P_x}{P_y} = \frac{20}{40} = \frac{1}{2}$$

So utility maximizing Charles should equate those two numbers. That is,

$$\frac{1}{X} = \frac{1}{2}$$

Since the budget constraint is

$$20X + 40Y = 120$$

and $X = 2$, we have $Y = 2$. Thus $(2, 2)$ is the optimal bundle that maximizes Charles? utility.

2.3 Multiple choice questions.

4. Danny is known to spend his entire income on cocktails (good X) and vinyl records (good Y). He has constant marginal utility for both. Which of the following statements must be true about Danny's consumption of cocktails and records? Assume that the marginal rate of substitution of X for Y is not equal to the price ratio.

(a) Danny will consume equal numbers of cocktails and records

(b) Danny will consume only cocktails

(c) Danny will consume only records

(d) Either (b) or (c)

Solution: The answer is (d). Danny will choose to consume more of X so long as $\frac{MU_X}{P_X} \geq \frac{MU_Y}{P_Y}$. With constant marginal utility of both X and Y, X and Y are perfect substitutes: if the MRS of X for Y is not equal to the price ratio, then it is either strictly greater than it or strictly less than it, regardless of how much Danny consumes of either good. Therefore, depending on the price ratio, Danny will consume only cocktails or only records.