

## Discussion 9 - Solutions

### Topics

- Long-Run Average Cost Curve
- Short- and Long-Run Equilibrium
- Long-Run Equilibrium

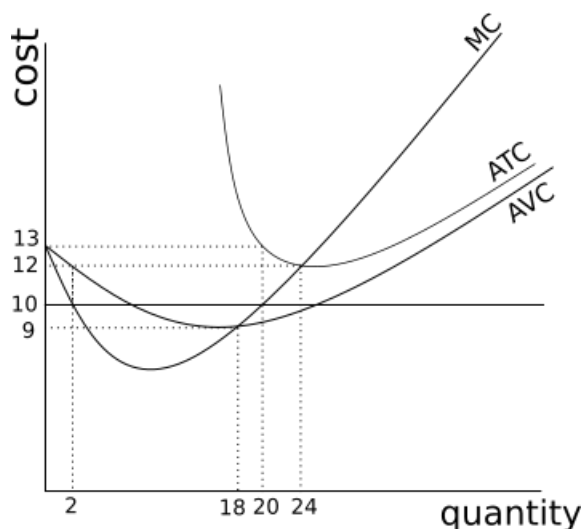
### Review: Firm Behavior in Perfectly Competitive Markets in Long-Run Equilibrium

1. Be able to define short run and long run for a production process.
  - *Short run:*
    - (a) *Cannot enter or exit an industry*
    - (b) *There are some factors of production (i.e. some types of inputs) that cannot be changed. For example, scale of factory, rental contracts, etc.*
    - (c) *Firms produce positive number of outputs when  $P >$  shutdown price.*
  - *Long run:*
    - (a) *Can enter or exit an industry*
    - (b) *All factors of production can be changed during long run. For example, scale of factory, rental contracts, etc.*
    - (c) *Firms produce positive number of outputs and stay in the market when  $P \geq$  break-even price.*
2. Define and the long run average cost curve for the representative competitive firm.  
*Long-run average cost curve is the lower envelope of the short-run average cost curves at different scales of production.*
3. Define and be able to identify economies to scale (or increasing returns to scale), diseconomies to scale (or decreasing returns to scale), and constant returns to scale.  
*We need to look at the average total cost function, and check how average total cost ATC varies with output level  $q$ .*
  - *If ATC increases, when  $q$  increases, we call it “diseconomies of scale.”*
  - *If ATC decreases, when  $q$  increases, we call it “economies of scale.”*
  - *If ATC does not change, when  $q$  increases, we call it “constant returns to scale.”*

- The minimum output level of ATC is called the Minimum Efficient Scale of the firm.

## Exercises

**Exercise 1 (Cost Curves and Equilibrium)** The graph below represents the cost structure for a firm that is in a perfectly competitive industry and a scale of production such that long run average costs are minimized:



1. What is the minimum price such that this firm will produce in the short-run?

*Solution:* In order for the firm to operate in the short-run, it must make enough revenue to at least cover its variable costs. Thus, the firm will only operate if price is at least equal to average variable cost (per unit). From the graph, this minimum price (the shutdown price) is \$9

2. What is the minimum price such that this firm will produce in the long-run?

*Solution:* For the firm to remain in the market in the long-run, the price would have to be at least \$12, the price at which marginal cost equals average total cost, i.e., the break-even price.

3. Suppose the total market demand in this industry is given by  $Q = 1000 - 20P$ . What is the long-run equilibrium price, number of firms, and units supplied by each firm? (Assume the number of firms has to be an integer)

*Solution:* From before, we know the break-even price is \$12, so this must be the long-run equilibrium price. Plugging this into the market demand, we get  $Q = 760$ . Finally, from the plot, each individual firm produces 24 units, thus, in order to meet market demand, there must be 31 firms in the market in the long-run (in

*fact there would be 31.67 firms, but we'll assume the number of firms has to be an integer).*

**Exercise 2 (Equilibrium in Taco Market)** Joe is one of many people in Madison who owns and operates a taco stand. Currently he and everyone else in town are offering essentially the same combinations of meat, cheese, guacamole and tortillas, and you really can't taste the difference between two different taco stands. Joe faces a variable cost of making tacos of  $VC = 10q + 1/2q^2$ , which gives him a  $MC = 10 + q$ . He has no fixed costs. All other stands have the same costs as Joe.

1. What is Joe's average total cost? Does Joe have economies of scale for tacos?

*Solution: ATC is  $TC/q$ . So  $ATC = (10q + 1/2q^2)/q = 10 + 1/2q$ . ATC is increasing with  $q$ , so there are not economies of scale (or you may say Joe has diseconomies of scale).*

2. Suppose the given market price is \$28 (these are really good tacos). What quantity does Joe supply at this price? *Solution: This is a perfectly competitive environment, so Joe sets price equal to marginal cost.  $P=MC$ , so  $28=10+q$ . Solving yields  $q = 18$ .*

3. If market demand is given by  $Q_D = 500 - 5P$ , how many tacos are sold in total? How many taco stands must be operating? What is the market supply curve?

*Plug in the price of \$28 into the demand curve to get  $Q_D = 500 - 5(28) = 360$ . Since all the firms are the same, they must each produce 18. Remember  $Q = Nq$ , so we have  $360 = 18N \rightarrow N = 360/18 = 20$ . So there are 20 taco stands.*

*Since all these firms are the same, we get market supply by multiply individual supply by 20. Remember that the individual supply curve is just marginal cost, i.e.  $P = 10 + q$ . Solve for  $q$  to get  $q = P - 10$ , then multiply by 20 to get  $20q = 20P - 200$ , so market supply is  $Q_S = 20P - 200$ .*

4. What is Joe's profit? Could this be a long run equilibrium in this perfectly competitive environment with free entry and exit? *Solution: Calculate profit in the usual way. Denote profit as  $\pi$ .  $\pi = TR - TC = pq - 10q - 1/2q^2$ . So profit is  $\pi = 28(18) - 10(18) - 1/2(18^2) = 504 - 180 - 162 = 162 > 0$ , so this cannot be a long run equilibrium.*

**Exercise 3 (Plastic chairs and Market Equilibrium)** The market for plastic chairs in Madison is perfectly competitive. The market demand for plastic chairs is given as  $P = 130 - Q$ . The market supply for plastic chairs is given as  $P = 2 + Q$ . Each firm faces the cost functions  $TC = 4q^2 + 2q + 64$  and  $MC = 8q + 2$ .

1. Determine the equilibrium quantity and price for this plastic chair market.

*Solution: Equating market demand and market supply,*

$$130 - Q = 2 + Q$$

$$\implies Q = 64, P = 66$$

2. What are the break-even price and the shutdown price for a representative firm in the long-run and short-run?

*Solution:*

*First we find the break-even price.*

$$ATC = MC \iff 4q + 2 + 64/q = 8q + 2$$

$$q = 4, P = 34$$

*Break-even price is \$34.*

*Now, we find the shutdown price.*

$$AVC = MC \iff 4q + 2 = 8q + 2$$

$$q = 0, P = 2$$

*Shutdown price is \$2.*

3. At the current equilibrium price, what is the quantity of chairs provided by a representative firm? Calculate a representative firm's profit.

*Solution: We solved that  $P = 66$ . Using the firm supply curve,  $MC = P = 8q + 2$ , we have*

$$66 = 8q + 2.$$

$$\implies q = 8.$$

*Profit is found by calculating  $TR - TC$ . For a firm,  $TR = 8 \times 66 = 528$ . We use the total cost function,  $TC = 4(8)^2 + 2(8) + 64 = 336$ . Thus,  $\text{profit} = \pi = 192$ .*

4. How many firms are in the market in the short-run?

*Solution: With market quantity  $Q = 64$  and firm quantity  $q = 8$ , there must be  $Q/q = 8$  firms.*

5. What is the long-run equilibrium price in the market? What is the long-run profit maximizing level of output for a representative firm? What is the long-run profit?

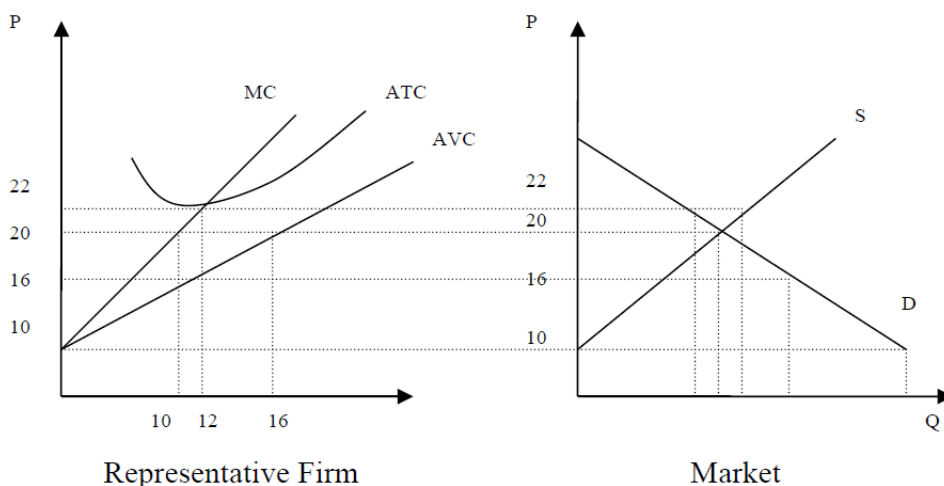
*Solution: In the long-run, the equilibrium price must be the break-even price,  $P = 34$ . Every firm will produce  $q = 4$  (from part 2). All firms earn zero profit.*

6. How many firms are in the market in the long run?

*Solution: Each firm must produce  $q = 4$ . With a market price of  $P = 34$ , we can find the market quantity by plugging this break-even (and therefore long run equilibrium price) into the market demand. Therefore,  $Q = 96$ . Now  $\# \text{firms} = 96/4 = 24$ .*

**Exercise 4** Use the following information to answer the next two questions.

Use the figures below to answer the next two questions:



1. What is the current market price and the industry's long-run equilibrium price? Is the industry currently in long-run equilibrium?
  - (a) \$20, \$22. Yes.
  - (b) \$16, \$16. Yes
  - (c) \$16, \$22. No
  - (d) \$20, \$22. No

*Solution: (d) Market price can be read off of the market equilibrium (graph on the right). The long-run equilibrium price is the minimum of long-run average cost of this representative firm, which is \$22 (also the break-even price here) according to graph on the left.*

*Since the current market price is lower than break-even price, in the long run, some firms exit, shifting up the market supply curve.*

2. Which of the following statements is correct?

- (a) Decrease or increase in consumer demand only changes the short-run equilibrium price, but not the long-run equilibrium price.
- (b) In the long run, the total quantity supplied will decrease.
- (c) In the long run, the quantity supplied by each supplier will increase.
- (d) All of the above.

*Solution: d.*

*a is correct because we know long-run equilibrium price is always pinned down by firms' minimum long-run average cost in a perfectly competitive industry with free entry and exit. In other words, only changes to firms' long-run average cost curves will change long-run equilibrium prices; changes to demand DO NOT change long-run equilibrium prices.*

*b is correct because right now firms are not breaking even, so some firms will exit, shifting up the supply curve. In the long run, equilibrium price will be at minimum of ATC, which is \$22, and the quantity supplied/demanded will be smaller than before.*

*c is correct because in the long run, equilibrium price is \$22, and each firm's production is 12 units (as opposed to 10 units currently).*

**Exercise 5** If profits are negative in the short-run in a perfectly competitive industry, which of the following would you *not* expect to happen as the market moves to the long run (assuming no external economies or external diseconomies of scale)?

- (a) The market price will increase.
- (b) Firms will exit the market.
- (c) Total market output will fall.
- (d) The demand curve faced by an individual firm will shift down.

*Solution: (d) In the long run, the price must rise. Each firm faces a perfectly elastic demand curve at the market price. Therefore, the quantity demanded of that firm's output will shift up. (Recall that the marginal revenue curve and the demand curve facing a competitive firm are identical).*

**Exercise 6** Suppose an industry currently has 10 identical firms. Assume their current scale of production is such that long run average costs are minimized. Their individual cost functions are:

$$\begin{aligned} \text{ATC} &= \frac{100}{q} + q \\ \text{MC} &= 2q. \end{aligned}$$

1. Given the information above, which of the following statements is correct?

- (a) The shutdown price is 0.
- (b) The break-even price is \$20.
- (c) The long-run equilibrium price is \$20.
- (d) All of the above.

*Solution: d.*

*Shutdown price: we know shutdown price happens where MC intersects AVC. We first need to back out AVC function.  $TC = ATC * q = 100 + q^2$ . Looking at this TC function, we know the only part that varies when  $q$  changes is  $q^2$ , so  $VC = q^2$ .  $AVC = \frac{VC}{q} = q$ . Set  $AVC = MC$ , we have  $q = 2q$ , and  $q=0$ .*

*Break-even price: we know break-even price happens where MC intersects ATC, so we set  $MC = ATC$ .  $2q = \frac{100}{q} + q \rightarrow q = \frac{100}{q} \rightarrow q^2 = 100$ , and we get  $q=10$ . But this is not the break-even price yet, to get break-even price, we plug in  $q=10$  back to MC function, or ATC function: break-even price  $= 2q = 10 * 2 = 20$ .*

*Long-run equilibrium price: \$20. See break-even price.*

2. Market demand is given by  $Q_D = 2000 - 15P$ . Which of the following statements is correct?

- (a) In the long run, some firms will exit.
- (b) In the long run, total quantity supplied will increase.
- (c) The industry is currently in long-run equilibrium.
- (d) None of the above.

*Solution: (b). The industry is currently not in long run equilibrium.*

*A roadmap: To check long-run equilibrium, we (1) first need to derive short-run market supply and (2) calculate short-run equilibrium price. Then we (3) compare short-run equilibrium price to the industry's long-run equilibrium price, which we get from previous question, \$20.*

*First, we know that 10 firms are in the market now, so  $Q_s = 10q$ . We also know short-run supply is given by marginal cost curve for each firm, so  $P = MC = 2q \rightarrow q = \frac{P}{2}$ . Combining these two equations, we have  $Q_s = 10q = 5P$ .*

*Next, we combine market supply  $Q_s = 5P$  with market demand,  $Q_D = 2000 - 15P$  to find out current equilibrium price and quantity:  $P_{sr} = 100$ ,  $Q_{sr} = 500$ .*

*Given that long-run equilibrium price should be \$20 (see previous part), and the current price is \$100, we know we are not in long run equilibrium. In particular,*

*we know that more firms will enter; total quantity supplied will increase from current quantity of 500 to 1700 (long run quantity:  $Q_D = 2000 - 15P = 2000 - 15 * 20 = 2000 - 300 = 1700$ ).*

**Exercise 7** Consider the market for headphones, which has many identical firms. Each competitive firm in this industry has cost functions as follows:

$$TC = 3q^2 + 4q + 48$$

$$MC = 6q + 4,$$

where quantity is in hundreds of thousands.

Assume that the current scale of production is such that long run average costs are minimized. The demand for tablets is given by

$$P = 88 - Q.$$

1. What is the long-run price?

- (a) \$26
- (b) \$27
- (c) \$28
- (d) \$29

*Solution: (c). We solve this problem by first using the breakeven condition, setting  $P = MC = ATC$ . From this we obtain that  $q = 4$ . Plugging this into our MC equation, we obtain  $P = \$28$ .*

2. In the long run, how many firms are there?

- (a) 12
- (b) 13
- (c) 14
- (d) 15

*Solution: (d). From the previous question, we obtain  $P = 28$ . To determine the number of firms, we can plug this into our demand equation to get  $28 = 88 - Q$ . But since  $Q$  is the industry supply, in order to solve for the total number of firms, we are looking for the  $N$  such that  $Q = N * q$ , or  $Q = 4N$ . This yields the equation  $60 = 4N$  and thus the solution  $N = 15$ .*

**Exercise 8** Assume a perfectly competitive industry is in long-run equilibrium, then the property taxes for firm increase



1. In the short run we expect that output levels:
  - (a) decrease for the firms and increase for the industry.
  - (b) decrease for the firms and decrease for the industry.
  - (c) remain unchanged for both the firms and the industry.
  - (d) increase for the firms and increase for the industry.
  - (e) increase for the firms and decrease for the industry.

*Solution: (c). Property taxes are a classic example of a fixed cost.*

*As we know, a change in fixed costs doesn't change output for the firm or industry in the short run. Because the short run impact of an increase in property taxes or any fixed cost is that total costs increase but since neither price nor marginal cost changes output doesn't change.*

2. The profits earned in the short run by firms after the property tax increase will:
  - (a) be identical to those earned before the increase since they still produce where  $P=MC$ .
  - (b) be slightly higher as firms increase price.
  - (c) be negative since production costs rose but price did not.
  - (d) be zero since price and cost will rise exactly the same amount.
  - (e) still be greater than zero, but slightly lower than before. increase for the firms and decrease for the industry.

*Solution: (C). Before property taxes increase, the perfectly competitive industry is in long-run equilibrium, so the profits for each firm were zero. After the the property taxes increase, profits will be negative since production costs rose but price did not.*

3. Over time the industry will adjust to a new long-run equilibrium. The new equilibrium will differ from the old in that:
  - (a) There will be fewer firms and market price will be higher.
  - (b) There will be fewer firms but market price will be the same.
  - (c) There will be fewer firms and market price will be lower.
  - (d) There will be more firms and market price will be higher.
  - (e) There will be more firms but market price will be the same.

*Solution: (a). The increase in fixed costs lead to short run losses which in turn lead to exit in the long run. This, in turn shifted back industry supply raising market price until it equaled the minimum of the new higher ATC curve.*