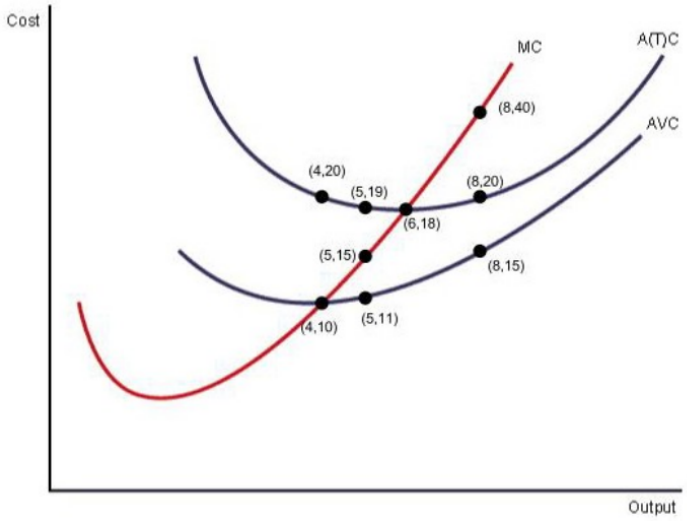
Discussion 8 - Solutions

Exercises

Exercise 1 (Cost Curves and Profit Maximization) Examine the following graph which shows the cost curves of an individual firm in the market for widgets, which is perfectly competitive:



1. What are the fixed costs of this firm?

   Solution: To find this, calculate total costs at a certain quantity and subtract total variable costs (by taking ATC and AVC and multiplying by quantity). You also could compute it in the opposite order by finding $AFC = ATC - AVC$ and then multiplying by quantity. For instance, at $q = 4$, $AVC = \$10$ and $ATC = \$20$. Thus $AFC = ATC - AVC = \$10$, so $FC = q \times \$10 = \$40$

2. How many widgets will the firm produce if the price is \$10?

   Solution: Using the firm's optimality rule ($MR = MC$) and the fact that the market is perfectly competitive (which implies that $P = MR$), the firm will produce where $MC = \$10$, i.e. at a quantity of 4.

3. What will be this firm's profit if the price is \$10?

   Solution: At $p = \$10 = MC$, $q = 4$, so $TR = 4 \times \$10 = \$40$. When the firm produces 4 units, $ATC = \$20$, so $TC = 4 \times 20 = \$80$. Thus the profit is $TR - TC = \$40 - \$80 = -\$40$. Note that the firm could also produce zero units, which would give a loss equal to the fixed costs of \$40.

4. Find the profit and the total variable cost of the firm when the widget price is \$15. Give an argument why the firm should not produce a quantity of 0 at this

price.

Solution: Profit $= TR - TC$. *We have* $TR = 5 \times \$15 = \$75$, *while* $TC = 5 \times \$19 = \$95$. *Thus profit* $= \$75 - \$95 = -\$20$. *Even if the firm produced a quantity of 0, it would still lose the fixed costs of \$40, which is a bigger loss, so it is better off producing* $q = 5$ *units than* $q = 0$ *(shutdown).*

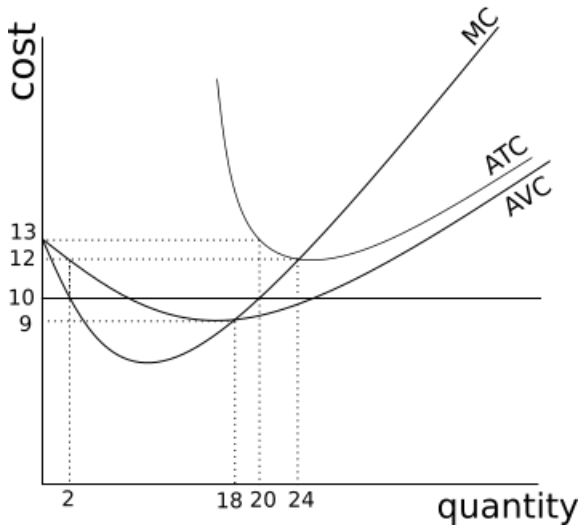
5. What is the firm's profit when the price is \$18?

   Solution: From looking at the diagram, when $P = \$18$, $q = 6$. *At this quantity ATC is also equal to \$18, so the firm makes zero profit.*

6. What is the firm's profit when the price is \$40?

   Solution: We have $TR = 8 \times \$40 = \$320$, *while* $TC = 8 \times \$20 = \$160$. *Thus profit is* $TR - TC = \$160$.

Exercise 2 The graph below represents the cost structure for a firm in a perfectly competitive industry:



1. Suppose the market price is \$10. Given the above information, what is the short-run profit-maximizing level of production for this firm?

   Solution: Our optimality condition is $MR = MC$, *which in the case of perfect competition reduces to* $P = MC$. *Looking at the graph, we have two potential intersections:* $q = 2$ *and* $q = 20$. *To see why* $q = 2$ *cannot be the optimal level of production, suppose the firm were producing two units. Then consider whether it would want to produce an additional unit. Because marginal cost falls below the price, that additional unit would be profitable, and thus* $q = 2$ *cannot be optimal. This leaves* $q = 20$ *as the only option. (An additional unit has marginal cost above the price, and thus would be unprofitable.)*

2. In the short-run, what is the firm's profit?

   Solution: Profit is given by $pq - TC(q) = pq - ATC(q)q$. *Revenue (pq) is* $\$10 \times 20 = \$200$. *From the graph average total cost per unit is \$13, so total costs are* $\$13 \times 20 = \$260$. *Thus the firm is operating at a loss of \$60.*

3. What is the minimum price such that this firm will produce in the short-run?

   Solution: In order for the firm to operate in the short-run, it must make enough revenue to at least cover its variable costs. Thus, the firm will only operate if price is at least equal to average variable cost (per unit). From the graph, this minimum price (the shutdown price) is \$9

Exercise 3 (Changes in Costs) Consider McKinsey & Company, a global consulting firm. Assume that McKinsey owns an office in NYC and produces strategic consulting papers by hiring Econ graduates. The market for consulting papers is perfectly competitive.

1. Suppose property taxes in NYC increase. Do total costs for McKinsey increase, decrease, or stay the same? What about variable costs?

   Solution: McKinsey owns its office, so it must pay property taxes on the space. In the short-run, McKinsey cannot sell its office, so these property taxes are a fixed cost. Then total costs will increase, and variable costs will stay the same.

2. Will McKinsey exit the market and shutdown production immediately? Why or why not?

   Solution: A firm will shutdown in the short-run if the minimum average variable costs are greater than marginal revenue. Because the property tax increase didn't change variable costs or marginal revenue, McKinsey will not shut down.

3. Will the number of consulting papers produced by McKinsey increase, decrease, or stay the same?

   A firm decides how much to produce by comparing marginal revenue and marginal cost. Neither marginal revenue or marginal costs have changed, so McKinsey will produce the same number of consulting papers.

Suppose that the property taxes are back to normal, and disregard any change that might have happened in the previous part. Now, the wage of Econ graduates employed in McKinsey has increased. After this change:

1. Do fixed costs for McKinsey increase, decrease, or stay the same? What about variable costs?

   Wages are a variable cost because McKinsey can change the number of workers it employs in the short-run. Then fixed costs stay the same and variable costs have increased.

2. Do we have enough information to decide whether McKinsey will shutdown in the short-run? Why or why not?

   We do not have enough information to decide if McKinsey will shutdown. The firm will shutdown if the minimum average variable cost is higher than the marginal revenue. Because wages have increased, average variable costs have increased, but we don't know if the minimum average variable cost is larger now than the marginal revenue. Then we can't say if McKinsey will shutdown or not.

Exercise 4 Find the short-run supply curve for a firm with $TC = 10 + 10\sqrt{q} + 5q^2$ and $MC = 10q + \frac{5}{\sqrt{q}}$.

1. What is the fixed cost for this firm? What is the variable cost?

   Solution: We find the fixed cost by finding total cost at q=0. This tells us that the fixed cost is 10. We know TC = TVC + TFC, so TVC = TC - FC. This tells us that the variable cost for this firm is $10\sqrt{q} + 5q^2$

2. What is the shutdown price for this firm?

   Solution: The shutdown price is found by solving AVC = MC.

$$TVC = 10\sqrt{q} + 5q^2$$

$$AVC = TVC/q = \frac{10}{\sqrt{q}} + 5q$$

$$\begin{aligned} AVC = MC &\iff \frac{10}{\sqrt{q}} + 5q = 10q + \frac{5}{\sqrt{q}} \\ &\iff \frac{5}{\sqrt{q}} = 5q \\ &\iff \frac{1}{\sqrt{q}} = q \\ &\iff q = 1 \end{aligned}$$

   Thus, the shutdown price is $P = 15$.

3. If the market price is P, which is greater than the shutdown price, how much will the firm produce?

   Solution: When price is greater than the shutdown price, the firm chooses production by setting MR=MC. Now we have supply $P = 10q + \frac{5}{\sqrt{q}}$ *for* $P \geq 15$ *and* $q = 0$ *for* $P < 15$.

Exercise 5 (Derive Supply Curve) Consider a bakery with the following cost functions in a perfectly competitive bread market:

$$TC = q^2 + 4q + 5$$

$$MC = 2q + 4$$

where q denotes pounds of bread.

1. What is the shutdown price for this bakery?

   Solution: Note that

$$AVC = \frac{VC}{q} = \frac{q^2 + 4q}{q} = q + 4.$$

Hence, at the shutdown price,

$$2q + 4 = MC = AVC = q + 4,$$

so $q = 0$. *By plugging this back into the MC function, we obtain the shutdown price:*

$$P_{sd} = 2 \times 0 + 4 = 4.$$

2. Derive the short-run supply curve.

Solution: The supply curve is $P = MC$:

$$Q = \begin{cases} \frac{P}{2} - 2 & \text{if } P \geq 4 \\ 0 & \text{if } P < 4 \end{cases}$$

Now suppose that there are 10 bakeries with the same cost functions in the bread market and that the market demand curve is given by $P = -Q_d + 40$

3. Find the short-run equilibrium in the bread market.

Solution: Note that for each $P \geq 4$, *the optimal output level for an individual bakery is*

$$q = \frac{P}{2} - 2.$$

Since all 10 firms have the same cost function, they have the same optimal output level. Hence the market supply curve is given by

$$Q_S = \begin{cases} 10q = 10(\frac{P}{2} - 2) = 5P - 20 & \text{if } P \geq 4 \\ 0 & \text{if } P < 4 \end{cases}$$

On the other hand, the market demand is

$$Q_D = 40 - P.$$

Hence at the equilibrium,

$$5P* - 20 = Q_S = Q_D = 40 - P*,$$

so

$$P* = \frac{60}{6} = 10.$$

By plugging this into the market demand cuve, we obtain

$$Q* = 40 - 10 = 30.$$

Therefore the equilibrium is (Q,P*)=(30,10).*

1 Multiple Choice Questions

Exercise 6 Generally speaking, and without looking at the numbers, let us consider the relationship between total revenue (TR), total variable cost (TVC), and the total cost (TC; the sum of total variable and total fixed costs). At which point should a firm consider shutting down? Can you draw the relationship between these curves and explain your answer?

(a) When TR is higher than TVC, but lower than the TC.

(b) When TR is lower than TVC.

(c) Never; as long as the firm is selling positive quantities of a good, it should continue its operations.

(d) When TR is exactly equal to TC.

Solution: (b) This is a point known as the shutdown point on a cost curve. This means that the firm will lose more money than the fixed cost of operations, and to minimize losses, should just shut down.

Exercise 7 Use the following information to answer the next two questions. Suppose that you start a business selling cases of lemonade for sporting events. Let the marginal cost curve for your lemonade business be given by the equation $MC = q^2 + 2$. The market price for a case of lemonade is \$27. You are a profit-maximizing firm.

1. How many cases of lemonade should you be selling?

   (a) 3

   (b) 5

   (c) 7

   (d) 9

   Solution: (b) You should be selling up to where $P = MC$, or 5 cases of lemonade.

2. If your average total cost at this quantity is \$20 per case, what is the profit you anticipate, if any?

   (a) 7

   (b) 35

   (c) 0

   (d) 12

   Solution: (b). Since you will make \$7 per cases, times 5 cases, you will get a profit of \$35.

Exercise 8 Cost Curves.

1. Average total cost equals average variable cost when:

   (a) The marginal cost is zero.

   (b) Marginal cost equals average variable cost.

   (c) Marginal cost equals average total cost.

   (d) Fixed costs are zero.

   Solution: We know that $ATC = AVC + AFC$. Then $ATC = AVC$ means that $AFC = 0$, which tells us that fixed costs are zero.Then the answer is (d).

2. Which of the following must be always true as the quantity for output increases?

   (a) Marginal cost must rise.

   (b) Average total cost must rise.

   (c) Average variable cost must rise.

   (d) Average fixed cost must fall.

   Solution: $AFC = FC/Q$. FC does not change as quantity increases, so AFC must fall, so (d) must always be true. Marginal can be increasing or decreasing in quantity, so we do not know if (a), (b), (c) are true.