Discussion 7

1 Review

1.1 Topics

- Graphing TPk, MPk, and APk

- Technology Choice

- Cost Curves and equilibrium production levels

- Profit Maximization

1.2 The Production Function

- A **production function** gives us the relationship between the quantity of inputs a firm uses and the quantity of output it produces. A firm can use **fixed inputs**, which are inputs whose quantities cannot be changed; and **variable inputs**, which are inputs that the firm can decide to use more or less of.

- The firm makes decisions about production in the **short run**, the time period in which at least one input is fixed, and in the **long run**, the time period in which all inputs can be varied.

- The **total product curve** shows how the quantity produced by the firm depends on the quantity of the variable input, for a given quantity of the fixed input.

- The **marginal product** of an input is the change in quantity of output produced by one additional unit of that input: $MPL = \frac{\text{change in quantity of output}}{\text{change in quantity of input}}$

1.3 Cost of Production

- A **fixed cost** is a cost that does not depend on the quantity of the output produced. This relates to the cost of the fixed input.

- A **variable cost** is a cost that depends on the quantity of the output produced. This relates to the cost of the variable input.

- The **total cost** of producing a given quantity of output is the sum of the fixed cost and the variable cost: $TC = FC + TVC$

- The **marginal cost** is the change in total cost generated by one additional unit of output: $MC = \frac{\text{change in total cost}}{\text{change in quantity of output}}$

- **Average total cost**, or **average cost**, is the total cost divided by the quantity of output produced: $ATC = \frac{TC}{Q}$

- **Average fixed cost** is the fixed cost per unit of outpout: $AFC = \frac{FC}{Q}$

- **Average variable cost** is the variable cost per unit of output: $AVC = \frac{VC}{Q}$

- Combining all of these, when the average total cost curve is U-shaped, average total cost is at its minimum at the bottom of the U. We call this point the **minimum-cost output**. In this point:

  - At the minimum-cost output, average total cost is equal to marginal cost.

  - At output less than the minimum-cost output, marginal cost is less than average total cost and average total cost is falling.

  - At output greater than the minimum-cost output, marginal cost is greater than average total cot and average total cost is rising.

***Looking forward: Our 'golden rule' for profit maximization in a competitive market is Marginal Revenue = Marginal Cost... a little rhyme "MR=MC gives me the Price and Quantity to produce most profitably"

2   Exercises

## 2.1   Graphing TPk, MPk, and APk

Nile, an online shopping company, has completely automated their shipping process and only uses robots to fill orders. The following table shows how many orders each robot can fulfill

Number of Robots	Number of Orders Filled	Marginal Product of Capital	Average Product of Capital
0	0	-	-
1	100	100	100
2	400	300	200
3	750	350	250
5	1500	375	300
10	3300	360	330
15	4500	240	300
25	4000	-50	160

1. Fill in the MPk and APk columns of the table.

2. Below, graph the Total Product of Capital Curve on the top graph. On the second graph, graph both the Marginal Product of Capital curve and the Average Product of Capital curve.

   Solution: (For total Product increase until Q=15, then decrease)

   (For Average Product/Marginal Product, Average will increase, at a decreasing rate until 10 then decline, Marginal Product will increase until 5 then decrease and cross Average Product just after 10)
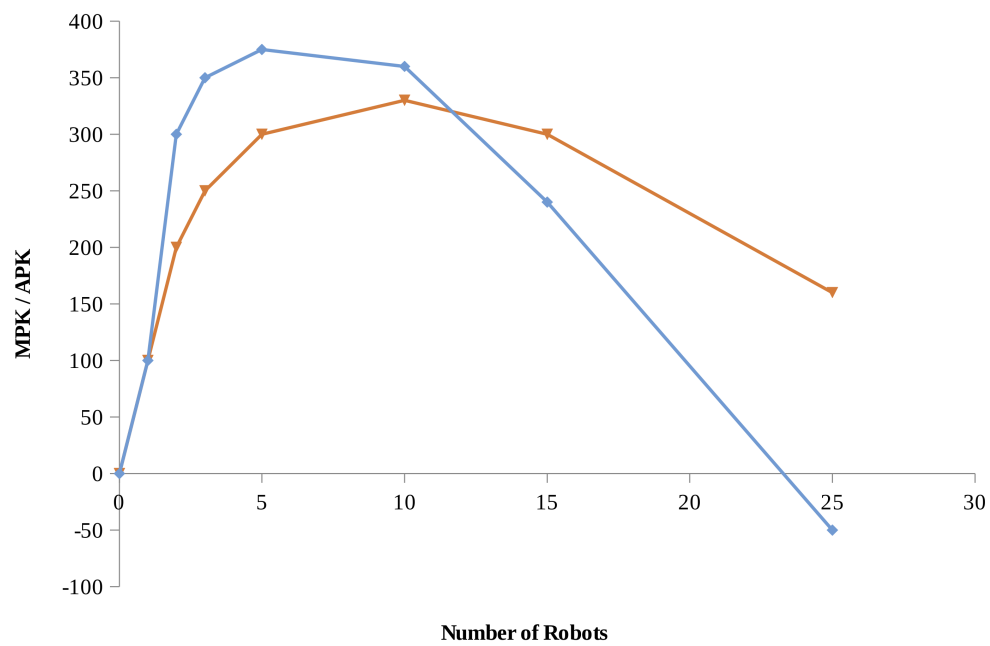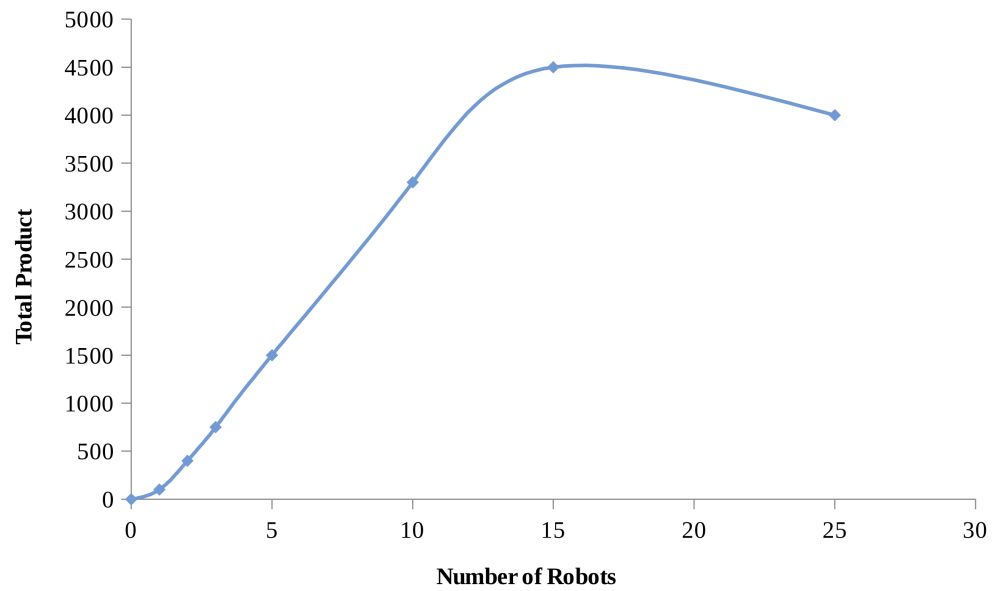
3. Over what number of robots are there increasing returns to capital? Diminishing returns to capital? Negative returns to capital? *Solution: Increasing returns: 0 to 5 robots; Diminishing returns: higher than 5 robots; Negative returns to scale: starts somewhere in between 15 and 25 robots and continues to be negative.*

4. General question: Given a typical production process, when average product (AP) increases, the marginal product (MP) *Increases and then decreases* .

2.2   Firm Choice of Technology

Suppose Babcock Dairy Factory has the following three technologies available to produce banana milk. All three produce the same amount of banana milk and combine labor and capital price in the proportions displayed in the table below:

Technology	Labor	Capital
A	4	3
B	2	5
C	4	4

1. Is there a technology that will never be used?
   Solution: Technology C will never be used. It is strictly inferior to technology A.

2. If the unit price of labor is $3 and the unit price of capital is $4, which technology would be used?
   Solution: The factory uses the technology with the least cost per unit of output. The cost of producing one gallon of milk by technology A is $4 \times 3 + 3 \times 4 = 24$ while by technology B it is $2 \times 3 + 5 \times 4 = 26$. Since $24 < 26$ technology A will be used.

3. If the unit price of labor is $4 and the unit price of capital is $2, which technology would be used?
   Solution: The factory uses the technology with the least cost per unit of output. The cost of producing one gallon of milk by technology A is $4 \times 4 + \times 2 = 22$ while by technology B is $2 \times 4 + 5 \times 2 = 18$. Since $22 > 18$, technology B will be used.

2.3   Firm cost functions and profit maximization

You are working in a small construction company in a perfectly competitive housing market. You accidentally spilled coffee on an important file below, which summarizes the company's costs.

Q	TC	VC	FC	ATC	AVC	AFC	MC
0	600	0	600	–	–	–	–
1	650	50	600	650	50	600	50
2	800	200	600	400	100	300	150
3	1800	1200	600	600	400	200	1000
4	3000	2400	600	750	600	150	1200
5	5000	4400	600	1000	880	120	2000

1. Restore the file by filling all the blanks.

2. Suppose your construction company is in a perfectly competitive market where the price is P=1200. What quantity should the firm produce in order to maximize its profit? What is the firm's profit at that price and quantity?
   Solution: By the price-taker and profit-maximization conditions, $P = MR = MC$. Hence, at $P = 1200$, the company will produce $Q = 4$. Therefore, the profit is $\pi = TR - TC = 1200 \times 4 - 3000 = 1800$.

2.4   Changes in Costs

Consider McKinsey & Company, a global consulting firm. Assume that McKinsey owns an office in NYC and produces strategic consulting papers by hiring Econ graduates. The market for consulting papers is perfectly competitive.

1. Suppose property taxes in NYC increase. Do total costs for McKinsey increase, decrease, or stay the same? What about variable costs?

   Solution: McKinsey owns its office, so it must pay property taxes on the space. In the short-run, McKinsey cannot sell its office, so these property taxes are a fixed cost. Then total costs will increase, and variable costs will stay the same.

2. Will McKinsey exit the market and shutdown production immediately? Why or why not?

   Solution: A firm will shutdown in the short-run if the minimum average variable costs are greater than marginal revenue. Because the property tax increase didn't change variable costs or marginal revenue, McKinsey will not shut down.

3. Will the number of consulting papers produced by McKinsey increase, decrease, or stay the same?

   A firm decides how much to produce by comparing marginal revenue and marginal cost. Neither marginal revenue or marginal costs have changed, so McKinsey will produce the same number of consulting papers.

Suppose that the property taxes are back to normal, and disregard any change that might have happened in the previous part. Now, the wage of Econ graduates employed in McKinsey has increased. After this change:

1. Do fixed costs for McKinsey increase, decrease, or stay the same? What about variable costs?

   Wages are a variable cost because McKinsey can change the number of workers it employs in the short-run. Then fixed costs stay the same and variable costs have increased.

2. Do we have enough information to decide whether McKinsey will shutdown in the short-run? Why or why not?

   We do not have enough information to decide if McKinsey will shutdown. The firm will shutdown if the minimum average variable cost is higher than the marginal revenue. Because wages have increased, average variable costs have increased, but we don't know if the minimum average variable cost is larger now than the marginal revenue. Then we can't say if McKinsey will shutdown or not.

3   Multiple Choice Questions

3.1

A producer has 4 identical machines and is employing 9 identical workers, where each worker is paired with a single machine. It is known that the producer does not work herself, and she is producing using all the machines and all the workers.

The total production per week **for each one of the machines** as a function of the number of workers is presented in the following table:

Number of workers	Total output
1	70
2	120
3	150
4	160
5	162

   I. The marginal product of the ninth worker is 30 units per week.

  II. The total production of the producer will increase by 50 units per week if she will get one more machine.

 III. The producer should give up 1 machine in exchange for a compensation of 70 units.

Which of the above statements are true?

a. Only I.

b. I and III.

c. II and III.

d. I and II.

   Solution:
Statement I: The producer has 4 machines and 9 workers, therefore she will assign 2 workers to each machine, but one of the machines will get a 3rd worker. That worker's marginal product will be $150 - 120 = 30$ units. Hence, statement I is true.

Statement II: If the producer will get an extra machine, then she will assign 2 workers to the first 4 machines and 1 worker to the 5th machine. Thus, one worker's marginal product will increase from 30 to 70, for an overall production increase of 40. Thus, statement II is false.

Statement III: If the producer will give up 1 machine, she can assign 3 workers to every remaining machine. The machine that gets sold has two workers with marginal products of 70 and 50. When they get reassigned to the remaining machines that have two workers each, their marginal product will decrease to 30. Thus, overall output will decrease by $(70-30)+(50-30) = 40+20 = 60$. Therefore, a compensation of 70 units per week are more than enough for the producer to be willing to give up one machine. This statement is true. (Note: this calculation can also be performed by subtracting the total output with 3 machines and 9 workers from the output with 4 machines and 9 workers)

The correct answer is (b).

3.2

Teslon is a profit-maximizing automobile firm that employs workers (labor) and machines (capital). Teslon has the following curves for its total product (TP), marginal product of labor, (MP_L), and marginal product of capital (MP_K):

$$MP_L = 2 + \frac{2}{\sqrt{L}}, \ L > 0$$

$$MP_K = 1 + \frac{1}{\sqrt{K}}, \ K > 0$$

1. Which of the following statements is **false**?

   a. The firm never experiences negative returns to labor or capital

   b. The firm never experiences diminishing marginal returns to labor or capital

   c. The firm never experiences increasing marginal returns to labor or capital

   d. The total production curve never slopes downward

   Solution: (B). Since L only enters MP_L and K only enters MP_K as the denominator of something that otherwise adds to the marginal product,

the marginal product of capital and labor are only decreasing in capital and labor, respectively. This is the definition of diminishing returns, so the firm always experiences diminishing returns and never experiences increasing returns. Since $\frac{2}{\sqrt{L}}$ and $\frac{1}{\sqrt{K}}$ cannot be negative, MP_L and MP_K cannot be negative. Thus, (a) is true. The total product curve only slopes downward if marginal product is negative, so (d) is also true.

2. Firms typically cannot control their capital in the short run, and Teslon is no different. Suppose the firm currently employs 3 units of labor and 1 unit of capital. If it wanted to increase output by exactly 3 automobiles, what should it do?

   a. Fire 1 worker

   b. Hire 1 worker

   c. Hire 2 workers

   d. Hire 3 workers

Solution: (b). Since the marginal product of labor only depends on Labor and the marginal product of capital only depends on capital, hiring one worker will change total product by marginal product of labor at the new number of employees. If the firm hires one worker, then the marginal product of labor will be equal to MP_L when $L = 4$. At $L = 4$, $MP_L = 3$. Thus, hiring one more worker will increase output by 3.

3.3

Given the following total cost function, derive the fixed cost.

$$TC = \frac{5}{q+1} + 5 + 5q + q^2$$

a. $4

b. $5

c. $8

d. $10

Solution: We are tempted to think that (b) is the correct answer because the second element in the TC is the number 5 which does not depend on q. However, if we plug $q = 0$ we get

$$TC(q = 0) = \frac{5}{0+1} + 5 + 5 \times 0 + 0^2 = 5 + 5 = 10$$

Thus the correct answer is (d).

3.4

Use the following information to answer the next two questions.
Elise and Ling decide to start a tutoring business. To start their business, they need to invest in buying 2 units of the most recent version of MacBook Pro which costs $2,500 for one unit. The cost of hiring tutors will be $25,000 per year, cost of office supplies $1,000 per year, and the cost for advertising their business will be $4,000 per year. The market interest rate is 20% and suppose that the value of the computers does not change over time. Assume that all costs are paid at the end of the year.

1. What is the least amount of revenue per year they must make in their first year to keep the business open beginenumerate[a.]

2. $30,000

3. $31,000

4. $35,000

5. $36,000

Solution: They must have decided to start the business because the expected total revenue is larger than or equal to total cost. So the least amount of revenue they must have expected is simply the amount equal to variable cost. Therefore, variable cost is 25,000+1,000+4,000 =30,000. The answer is (a).