Discussion 6 Solutions

Today

- Utility maximization
- Substitution and Income Effects
- Demand Curve Derivation
- Engel Curve (and its derivation)

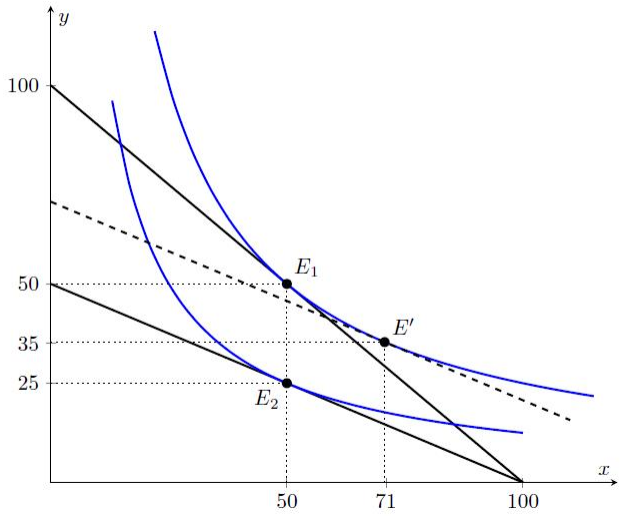
1 Problems

Exercise 1 (Consumer Theory Without Graphs) Bucky is a utility maximizer. Bucky has \$80 to spend on Badger basketball tickets and economics textbooks. Given that the price of basketball tickets is \$10 and economics textbooks is \$10 in addition to the data in the table:

Quantity	TU tickets	MU tickets	MU/\$	TU econ texts	MU econ texts	MU/\$
1	25	25	2.5	40	40	4.0
2	45	20	2.0	70	30	3.0
3	60	15	1.5	95	25	2.5
4	70	10	1.0	115	20	2.0
5	75	5	0.5	130	15	1.5
6	77	2	0.2	140	10	1.0
7	76	-1	-0.1	145	5	0.5

1. How many tickets and textbooks should Bucky purchase?
   Solution: Bucky should buy 3 tickets and 5 textbooks.
2. What is Bucky's total utility from the purchase?
   Solution: TU is 190.
3. What is Bucky's saturation point for basketball tickets?
   Solution: Bucky is saturated at 6 tickets. We do not know his saturation point for econ textbooks.

Exercise 2 Suppose 2Chainz has preferences between two chains (x) and (y), as represented in the graph below. Point E1 is 2Chainz's initial consumption bundle. Then, the price of chain 'y' increases, causing 2Chainz to consume at point E2. Point E' is the intermediate point between these two consumption bundles that is used to decompose the price (total) effect into income and substitution effects(it is therefore sometimes called a decomposition basket).



1. Will the substitution effect for chain 'x' be positive or negative? What about the income effect? Which effect is dominant? Explain.
   Solution: Substitution is the movement along the original indifference curve from E1 to E'. This is a positive effect. The income effect is the movement from E' to E2. This is a negative effect. The combined effects means 2Chainz consumes the same number of 'x' chains, so neither is dominant.

2. Will the substitution effect for chain 'y' be positive or negative? What about the income effect? Which effect is dominant? Explain.
   Solution: For chain 'y', we analyze the two effects by looking at the changes between the three points along the y axis. The substitution effect is from E1 to E' and the income effect is from E' to E2. We see that both effects are negative. The substitution effect is larger than the income effect.

3. Compute the magnitude of these effects by completing the table below.

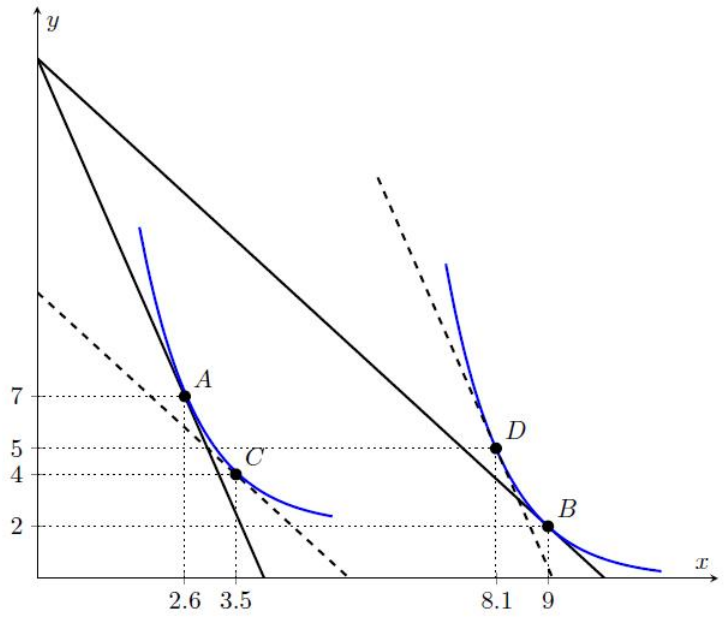
	Substitution effect	Income effect	Price (total) effect
Pointwise	E_1 to E'	E' to E_2	E_1 to E_2
Chain 'x'	$71 - 50 = 21$	$50 - 72 = -21$	0
Chain 'y'	$35 - 50 = -15$	$25 - 35 = -10$	-25

4. Assuming a linear demand curve for chain 'y', what will this demand curve look like?
   Solution: The demand curve for y will be downward sloping.

5. For 2Chainz is chain 'x' a normal or inferior good? What about chain 'y'? Justify your answer.

Solution: Both goods are normal because the income effects are negative when income is reduced (shift of budget constraint towards the origin).

Exercise 3 Professor Hansen has recently taken a liking to 1970's R&B/Soul music. She has preferences for Al Green albums (X), and Stevie Wonder albums (Y), which are represented in the graph below. Use it to answer the following questions.



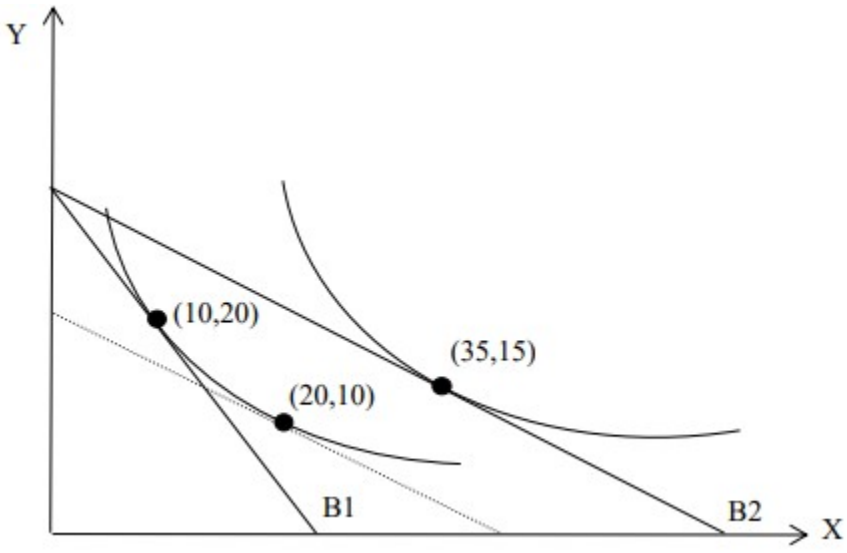
1. Are Al Green albums (X) a normal or inferior good for Prof. Hansen? What about Stevie Wonder albums (Y)? Explain.
   Solution: For this we want to look at the parallel shifts of the budget constraint - i.e. what happens between points A and D, or points C and B. Going from D to A, would be the income effect as a result of a price increase, and going from C to B would be the income effect resulting from a price decrease. Moving from point D to point A (price increase), we see that the quantity change in (X) is negative and the income change is also negative, therefore good x is a normal good. Between these same two points, the quantity change in (Y) will be positive and income change is negative, so this will be an inferior good.

2. Suppose Prof. Hansen is initially consuming at point A. Then the price of Al Green albums (X) decreases, causing her to consume at point B. To determine income and substitution effects, which intermediate point should be used to yield the correct result? C or D? Are both okay? (Hint: old utility, new prices.)
   Solution: The substitution effect will be from A to C. The income effect will be from C to B. Both points are NOT okay. We start by moving along the original utility, and then do the parallel shift.

3. Suppose Prof. Hansen is initially consuming at point B. Then the price of Al Green albums (X) increases, causing her to consume at point A. To determine income and substitution effects in this case, which intermediate point should be used to yield the correct result? C or D? Are both okay? (Hint: old utility, new prices.)
   Solution: The substitution effect will be from point B to D. The income effect will be from point D to A. Again both points are not okay, we need to follow this particular order.

2 Multiple Choice Questions

Exercise 4 Use the following information to answer the next two questions. Alice consumes only goods X and Y. The following graph represents Alice's utility maximization problem following an decrease in the price of X, changing her budget line from B1 to B2.



1. Which of the following statements describes Alice's income and substitution effects given the above information and graph?
   (a) Income effect is 5 units of Y; and Substitution effect is 10 units of Y.
   (b) Income effect is -10 units of Y; and Substitution effect is 5 units of Y.
   (c) Income effect is 10 units of Y; and Substitution effect is -5 units of Y.
   (d) Income effect is 5 units of Y; and Substitution effect is -10 units of Y.

   Solution: The answer is (d). Alice is originally consuming $(10, 20)$ *and after the price change is consuming* $(35, 15)$*. Since* $(20, 10)$ *is on the same indifference*

curve as $(10, 20)$ *and is on a budget constraint that is parallel to the new constraint, the substitution effect in Y can be calculated by subtracting 20 from 10. Also, the parallel shift of the budget constraint represents a change in income with prices kept the same. The two points on each line,* $(20, 10)$ *and* $(35, 15)$*, show the effects of an increase in income due to the price change. So, the income effect in Y is* $15 - 10 = 5$*.*

2. Now suppose that the initial price of good X is $P_{X1} = \$2$, the new price is $P_{X2} = \$1$, and the price of good Y is $P_Y = \$3$. Suppose that in order to purchase X with the lower price, Alice has to buy a coupon. What is the maximum amount that Alice would be willing to pay for that coupon? (Hint: The greatest amount would be when she retains her old utility level with the new price)

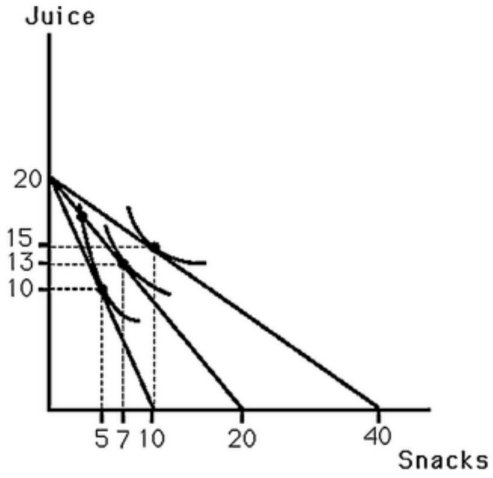
   (a) \$100

   (b) \$80

   (c) \$50

   (d) \$30

   Solution: The answer is (d). Note that Alice's income stays the same throughout this question. That is, her income was $2 \cdot 10 + 3 \cdot 20 = 80$ *on* $B1$ *and it's the same on* $B2$*. Alice would be willing to buy a coupon that will enable her to purchase X with the new, lower, price, since this will still increase her utility level - up to a point. That point would be when she buys the coupon but still has the same level of utility as with the former price. We know that the consumption bundle with the new price and the initial utility is* $(20, 10)$*. Alice's income with this bundle and the new price is* $1 \cdot 20 + 3 \cdot 10 = 50$*. Therefore, she would be willing to pay a maximum amount of* $\$80 - \$50 = \$30$ *for the coupon.*



Exercise 5 The above figure shows Lily's indifference curves for juice and snacks. Also

shown are three budget lines resulting from different prices for snacks. She has $20 to spend on these goods. Which one of the following point is on Lily's demand curve for snacks?

(a) p=$2, q=7

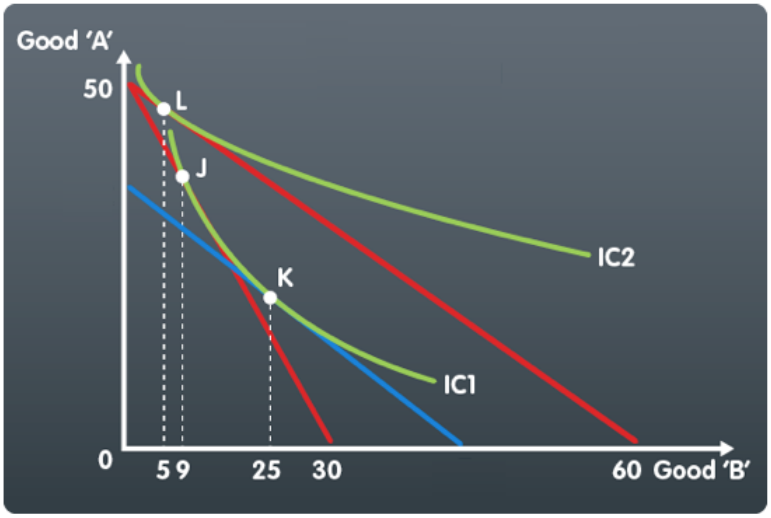
(b) p=$2, q=10

(c) p=$1, q=7

(d) p=$1, q=10

Solution: (c) is the answer because it is the only option that has Lily maximizing her utility under her budget constraint. Having $1 as price corresponds to the budget constraint with the x-intercept 20. With this budget constraint, Lily's maximizes her utility at (7,13).

Exercise 6 Use the following information to answer the next two questions. The following graph[1] describes the situation where the price of good B went down. IC1 indicates the indifference curve before the price change and IC2 indicates the indifference curve after the price change.



1. Which of the following statements describes the income effect?

   (a) The income effect is -20 units of Good B.

   (b) The income effect is -16 units of Good B.

   (c) The income effect is -4 units of Good B.

[1]Graph from Econfix (https://econfix.wordpress.com/2016/09/23/indifference-curves-and-giffen-goods/)

(d) The income effect is 16 units of Good B.

Solution: The answer is (a). The parallel shift of the budget constraint represents a change in income with prices being the same. The income effect is the change in good B moving from point K to L. Thus, the income effect is 5-25=-20 units of Good B.

2. Which of the following statements is/are true?

I. The price elasticity of demand is negative.

II. The price elasticity of demand is positive.

III. The income elasticity of demand is negative.

IV. The income elasticity of demand is positive.

(a) I and III only

(b) I and IV only

(c) II and III only

(d) II and IV only

Solution: The answer is (c). As the price of Good B decreases, the quantity demanded of Good B also decreases. So the price elasticity of demand is positive. From the previous question, we know that the income effect is negative. So the income elasticity of demand is negative.