Discussion 5

1 Topics

- Budget Constraints
- Utility Maximization
- Midterm 1 - Discussion of most missed questions

2 Concept Review

2.1 Budget Constraints

The budget constraint describes how a household or consumer can spend their income and/or wealth, given product prices. It defines the choice set/opportunity set, which is all the spending combinations one can make, while limited by the budget.

- **Formula:** When we have two goods, the budget constraint can written as

$$P_X X + P_Y Y = I$$

  where X is the number of good X consumed at price P_X, X is the number of good Y consumed at price P_Y, and I is income.

- **Changes:** When we have two goods

  1. Changes in price (P_X, P_Y) cause the budget constraint to pivot.
  2. Changes in income (I) lead to parallel shifts of the budget constraint.

2.2 Utility Maximization

Household and/or consumer preferences are measured by utility, or satisfaction. We assume everyone makes choices to maximize their utility.

- **Total utility:** The amount of utility gained from total goods or services. For example, if I eat 10 cookies, total utility is how satisfied I feel after eating all 10 cookies.

- **Marginal utility**: The amount of utility gained from each additional unit of goods or services. For example, marginal utility is how satisfied I feel after one additional cookie.

  - **Formula:** Marginal utility can be calculated from $\frac{\Delta TU}{\Delta Q}$, i.e. change in total utility over change in quantity of goods or services.

- **Law of Diminishing Marginal Utility:** The more of one good or service is consumed, the less marginal utility is gained from each additional unit of consumption. For example, the more cookies I eat, the less I feel satisfied with every additional cookie.
- **Marginal Rate of Substitution (MRS):** Given goods X and Y, MRS $= \frac{MU_X}{MU_Y}$ tells us the rate at which one is willing to substitute good X for good Y.
- **Utility Maximization Rule:** For a utility maximizing consumer, this must hold: $MRS = \frac{MU_X}{MU_Y} = \frac{P_X}{P_Y}$. The equation can be rearranged as $\frac{MU_X}{P_X} = \frac{MU_Y}{P_Y}$.

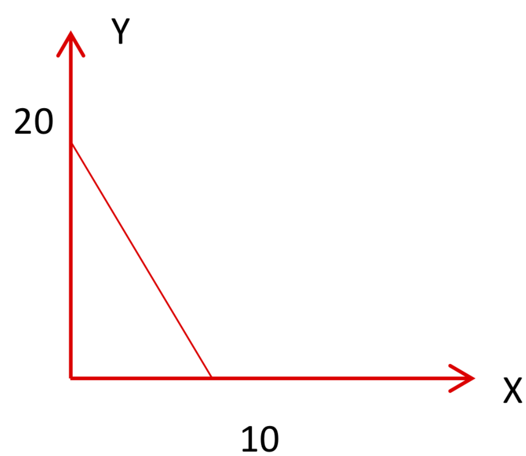
3 Exercises

3.1 Budget Constraint.

1. Suppose Michelle's income is \$100 per week and she only consumes oranges (good X) and apples (good Y). Each apple costs \$5 and each orange costs \$10.

1. Find the equation for Michelle's budget constraint and graph it.
   Solution:

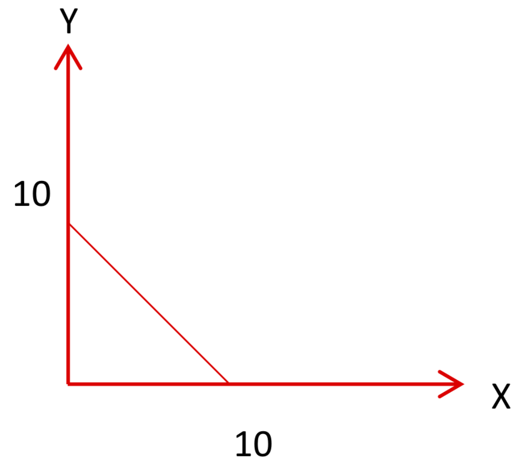
$$10X + 5Y = 100$$



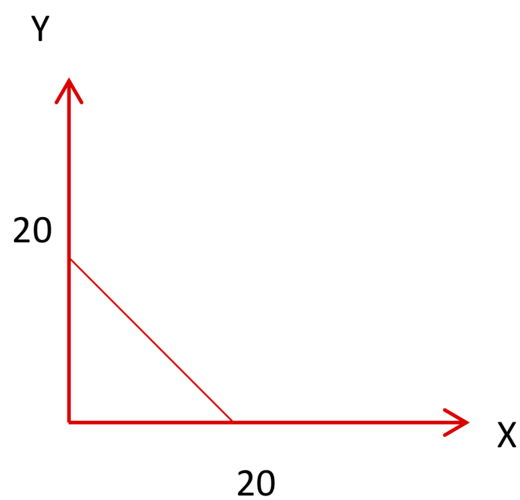
2. Is the bundle $(7, 6)$ affordable for Michelle? What about the bundle $(7, 7)$?
   Solution: Since the bundle $(7, 6)$ costs $\$70 + \$30 = \$100$, it is affordable. However, the bundle $(7, 7)$ costs \$105. So it is not affordable.

3. Now suppose that price of an apple increases from \$5 to \$10. Graph Michelle's new budget constraint and find the equation.
   Solution:

$$10X + 10Y = 100$$



4. Michelle got a raise and her income is now \$200. Assume the price of an apple is still \$10. Find the equation for Michelle's new budget constraint and graph it.
Solution:
$$10X + 10Y = 200$$



3.2 Utility Maximization.

Mark spent \$80 to buy 10 cans of beer (good X) and 5 bottles of wine (good Y) last week. Each can of beer cost P_x and each bottle of wine cost P_y. Suppose he is a utility maximizing agent.

1. Now, the price of one bottle of beer has increased by \$1 and he has spent the same amount of money to buy 8 cans of beer and 5 bottles of wine. Find the price of beer before the change and the price of wine.

Solution: We have two budget equations:

$$10P_x + 5P_y = 80$$

$$8(P_x + 1) + 5P_y = 80$$

By subtracting the second equation from the first equation, we get $2P_x = 8$. *So,* $P_x = 4$. *Plugging* $P_x = 4$ *in one of any budget equations gives* $P_y = 8$ *Thus* $P_x = 4, P_y = 8$.

2. What is the MRS before the price change and after the price change?

   Solution: Before the change:

$$MRS = \frac{MU_X}{MU_Y} = \frac{P_x}{P_y} = \frac{4}{8} = \frac{1}{2}$$

   After the change:

$$MRS = \frac{MU_X}{MU_Y} = \frac{P_x + 1}{P_y} = \frac{4+1}{8} = \frac{5}{8}$$

3. Charles derives utility from pairs of black shoes (good X) and pairs of blue jeans (good Y). The marginal utility of a pair of black shoes is $MU_X = \frac{1}{X}$. The marginal utility of a pair of blue jeans is 1. He has an income of $120. Suppose a pair of black shoes costs $20, and a pair of blue jeans costs $40. Which bundle should he consume if he wants to maximize his utility?

   Solution: MRS of black shoes for blue jeans is

$$MRS = \frac{MU_X}{MU_Y} = \frac{1}{X}$$

   The price ratio between the two goods (or the slope of the budget constraint) is

$$\frac{P_x}{P_y} = \frac{20}{40} = \frac{1}{2}$$

   So utility maximizing Charles should equate those two numbers. That is,

$$\frac{1}{X} = \frac{1}{2}$$

   Since the budget constraint is

$$20X + 40Y = 120$$

   and $X = 2$, *we have* $Y = 2$. *Thus* $(2, 2)$ *is the optimal bundle that maximizes Charles' utility.*

3.3 Multiple choice questions.

1. Danny is known to spend his entire income on cocktails (good X) and vinyl records (good Y). He has constant marginal utility for both. Which of the following statements must be true about Danny's consumption of cocktails and records? Assume that the marginal rate of substitution of X for Y is not equal to the price ratio.

   (a) Danny will consume equal numbers of cocktails and records

   (b) Danny will consume only cocktails

   (c) Danny will consume only records

   (d) Either (b) or (c)

   Solution: The answer is (d). Danny will choose to consume more of X so long as $\frac{MU_X}{P_X} \geq \frac{MU_Y}{P_Y}$. With constant marginal utility of both X and Y, X and Y are perfect substitutes: if the MRS of X for Y is not equal to the price ratio, then it is either strictly greater than it or strictly less than it, regardless of how much Danny consumes of either good. Therefore, depending on the price ratio, Danny will consume only cocktails or only records.

2. Katherine is deciding on how many chocolate chip cookies versus oatmeal raisin cookies to eat. Her marginal utility for chocolate chip cookies is 30, and her marginal utility for oatmeal raisin cookies is 2. Suppose the price of chocolate chip cookies is \$20, and the price of oatmeal raisin cookies is \$2. Then at this point, Katherine must be consuming:

   (a) Too many oatmeal raisin cookies

   (b) Too many chocolate chip cookies

   (c) The right amount of both cookies to maximize her utility

   Solution: The answer is (a). Given the information, we can find that $MRS = MU_{choc}/MU_{oat} = 30/2 = 15$. And $P_{choc}/P_{oat} = 20/2 = 10$. Since $15 > 10 \implies MRS > P_{choc}/P_{oat}$, Katherine is should consider more chocolate chip cookies and less oatmeal raisin cookies to reach the utility maximizing point. As she consumes more chocolate chip cookies, her MU_{choc} will decrease, and vice versa for oatmeal raisin cookies. Therefore, Katherine is consuming too many oatmeal raisin cookies at the point described in the question.