Discussion 4

1 Topics

- Elasticity

2 Elasticity Problems

Exercise 1

Suppose demand is given by $Q = 10 - P$.

1. Suppose the price is originally at \$4 and increases to \$5. Using these two points, and the regular percentage change formula, calculate price elasticity of demand.

2. Now suppose the price falls from \$5 to \$4. Again using the regular percentage formula, calculate price elasticity of demand. What problem is apparent given your answers to this and the previous part?

   Solution to (1)&(2): If price is \$4, then quantity is 6, and similarly if price is \$5, quantity is 5. So recalling the regular percentage formula for price elasticity of demand

   $$E_d = \left(\frac{Q_1 - Q_0}{Q_0}\right) / \left(\frac{P_1 - P_0}{P_0}\right)$$

   So price elasticity of demand from \$4 to \$5 is $-2/3$.
   Repeating this for part (2) but in the opposite direction, we find price elasticity of demand to be -1. Clearly this is problematic since, using the same price points, we found different elasticities simply by changing the order of the price change.

3. Using the mid-point formula, calculate the price elasticity of demand between \$4 and \$5. What is the advantage of this formula? Is demand elastic or inelastic?

4. Calculate the revenue to a firm in this market if the price is set at \$4 and \$5.

   Solution to (3) & (4): Recall the mid-point formula is

   $$E_d = \left(\frac{Q_1 - Q_0}{(Q_1 + Q_0)/2}\right) / \left(\frac{P_1 - P_0}{(P_1 + P_0)/2}\right)$$

   So using the prices from above we now find mid-point price elasticity of demand to be $-9/11$. Since elasticity is greater than -1, we conclude that demand is inelastic in this range.
   If the price is \$4, 6 units are sold, so revenue would be \$24. Similarly is the price is \$5, revenue is \$25. Notice, increasing the price, resulted in increased revenue.

5. Repeat parts (3) and (4) using the prices \$8 and \$9. Make a conjecture about the relationship between price elasticity and the effect of increasing price on revenue. What will be the elasticity at the revenue maximizing price?

Solution: Repeating the process for price of \$8 and \$9, we find mid-point price elasticity of demand to be $-17/3$. Since elasticity is smaller than -1, we conclude that demand is elastic in this range.
Computing revenue as before, we find revenue with a price of \$8 is \$16, and revenue with a price of \$9 is \$9. So here increasing the price resulted in decreased revenue.
We could conjecture that increasing the price causes increased (decreased) revenue if the curve is inelastic (elastic) in that price range. Thus, noticing that elasticity is increasing as prices increase, to maximize revenue, we should raise prices until the curve just becomes elastic, i.e. when price elasticity of demand is precisely equal to -1.

Exercise 2

The following table gives part of the demand schedule for widgets in the USA:

Price	1	3	5
Quantity Demanded	18	10	4

1. Calculate the price elasticity of demand as the price increases from \$1 to \$3 using the mid-point formula. What happens to the total revenue as a result of the price increase?

   Using the midpoint elasticity formula, we have :

   $$E_d = \frac{\frac{10-18}{(10+18)/2}}{\frac{3-1}{(3+1)/2}} = -\frac{4}{7}$$

   Therefore, E_d is greater than -1, so demand is inelastic on this portion of the demand curve. Hence, an increase in price will increase total revenue.

2. Calculate the price elasticity of demand as the price increases from \$3 to \$5 using the mid-point formula. What happens to the total revenue as a result of the price increase?

   Using the midpoint elasticity formula, we have :

   $$E_d = \frac{\frac{4-10}{(4+10)/2}}{\frac{5-3}{(5+3)/2}} = -\frac{12}{7}$$

   Therefore, E_d is less than -1, so demand is elastic on this portion of the demand curve. Hence, an increase in price will decrease total revenue.

Exercise 3

Suppose demand for Donuts in Madison is given by $P = 120 - 3Q$

1. Calculate the price elasticity of demand at $P = 30$. What adjustment should be made to the price to increase total revenue?

   Plugging $P = 30$ into the demand equation gives $Q = 30$. We can now calculate the elasticity at $P = 30$ using the slope formula,

   $$E_d = \frac{1}{m} \cdot \frac{P}{Q} = -\frac{1}{3} \cdot \frac{30}{30} = -\frac{1}{3}$$

   Since E_d is greater than -1, the demand is inelastic at $P = 30$. Therefore, we should increase price to achieve a higher level of total revenue (but not higher than the price at which the elasticity is -1).

2. Calculate the price elasticity of demand at $P = 90$. What adjustment should be made to the price to increase total revenue?

   Plugging $P = 90$ into the demand equation gives $Q = 10$. We can now calculate the elasticity at $P = 90$ using the slope formula,

   $$E_d = \frac{1}{m} \cdot \frac{P}{Q} = \frac{1}{3} \cdot \frac{90}{10} = -3$$

   Since E_d is less than -1, the demand is elastic at $P = 90$. Therefore, we shoule decrease price to achieve a higher level of total revenue (but not lower than the price at which the elasticity is -1).

3. Calculate the price and quantity at which the total revenue is maximized. What is the maximum revenue that can be obtained?

   The total revenue is maximized at the point where $E_d = -1$. Plugging in the slope formula for E_d, we have

   $$E_d = \frac{1}{m} \cdot \frac{P}{Q} = -\frac{1}{3} \cdot \frac{P}{Q} = -1$$

   And don't forget we have demand curve, which gives us the relationship between P and Q! Therefore we can plug in $P = 120 - 3Q$ into the equation above, and get an equation in terms of Q only :

   $$-\frac{1}{3} \cdot \frac{P}{Q} = -\frac{1}{3} \cdot \frac{120 - 3Q}{Q} = -1$$

   Thus $Q = 20$.

3 Multiple Choice Questions

Exercise 4 Which of the following statements is true?

(a) The price elasticity of demand for downward-sloping linear demand is constant across all points on the demand curve.

(b) With a perfectly elastic demand curve, increasing price increases total revenue.

(c) With a unitary elastic demand curve, increasing price increases total revenue.

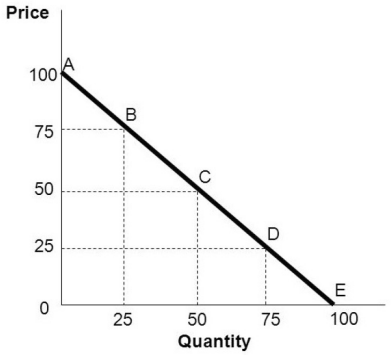
(d) On the inelastic part of a demand curve, increasing price increases total revenue.

Answer: (d).

(a) is incorrect because for a downward sloping linear demand, the price elasticity decreases, as we move down the demand curve (or move from left to right). (b) is incorrect, because the demand elasticity is negative infinity everywhere on the perfectly elastic demand curve. (c) is incorrect, because with a unitary elastic demand curve, the total revenue does not change when you increase or decrease price. (d) is correct.

Exercise 5 Which of the following statements is true?

Table 1: Multiple Choice Question 2



(a) Point B is more elastic than point D.

(b) At point C, elasticity is 0.

(c) All points are elastic.

(d) None of the above.

Answer: (a).
(a) is correct; you can show it by calculating elasticities at point B and at point D. (b) is incorrect, 0 elasticity would mean perfectly inelastic demand, which should be a

vertical line. At point C, elasticity is -1. (c) is incorrect, because any point below point C has elasticity greater than -1, and by definition, inelastic.

Exercise 6 Consider the market for peanut butter and jars of jelly. Suppose that the price elasticity of demand for peanut butter is -2, and that there is a 10% drop in the quantity demanded for peanut butter following a change in price. What is the price change?

(a) A 10% decrease in price

(b) A 5% decrease in price

(c) A 10% increase in price

(d) A 5% increase in price

Answer: (d).
We know that for a 1% increase in price, the demand drops 2%; assuming demand is linear, then for a 10% drop in demand, the price will have increased a corresponding 5%. ($E_d = -2 = \frac{-10\%}{\Delta p/p}$, i.e. $\frac{\Delta p}{p} = 5\%$.)

Exercise 7 Following from the previous exercise, now suppose that the cross-price elasticity of demand for jelly in response to a change in price for peanut butter is -3. What is the change in demand for jelly?

(a) A 15% decrease in the demand for jelly

(b) A 5% decrease in the demand for jelly

(c) A 15% increase in the demand for jelly

(d) A 5% increase in the demand for jelly

Answer: (a).
Given that the previous problem gave a 5% increase in the price of peanut butter, we must multiply the cross-price elasticity for jelly by 5% to get the corresponding change in the demand for jelly.
cross-price elasticity of demand for jelly $= \frac{\Delta Q_{jelly}}{Q_{jelly}} / \frac{\Delta P_{peanut\ butter}}{P_{peanut\ butter}} = -3$,
i.e. $\frac{\Delta Q_{jelly}}{Q_{jelly}} = -3 \times \frac{\Delta P_{peanut\ butter}}{P_{peanut\ butter}} = -3 \times 5\% = -15\%$.

Exercise 8 Alice is a book lover and she says, "I always spend half of my monthly salary on books regardless of the price." We can conclude that her demand for books is

(a) elastic

(b) inelastic

(c) unitary elastic

(d) perfectly inelastic

Answer: (c).
Since the total expenditure of Alice (total revenue of book sellers) will remain unchanged if the price increases or decreases, Alice has a unitary elastic demand.

Exercise 9 Suppose the government imposed an effective price floor on a good. If the government raises the effective price floor, the total revenue

(a) would increase

(b) would decrease

(c) would remain unchanged

(d) may increase, decrease, or remain unchanged

Answer: (d).
Under the effective price floor, the quantity transacted is equal to the quantity demanded. Because of a higher price, the quantity transacted drops. Since total revenue is the product of the market price and quantity transacted, whether total revenue rises or falls depends on the price elasticity of demand. For example, if the demand is inelastic, the total revenue would increase.

Exercise 10 Suppose the government imposed an effective price ceiling on a good. If the government raises the effective price ceiling, the total revenue

(a) would increase

(b) would decrease

(c) would remain unchanged

(d) may increase, decrease, or remain unchanged

Answer: (a).
Under the effective price ceiling, the quantity transacted is equal to the quantity supplied. Because of a higher price, the quantity transacted increases. So the total revenue would increase since it is the product of the market price and quantity transacted.