Discussion 13

Topics

- Game Theory
- Monopolistic Competition

Exercises

Exercise 1. Game Theory

King Robert is the ruler of the seven kingdoms. His kingdoms trade with the nine free cities of Essos. King Robert suspects that his kingdoms don't have the upper hand in this bilateral trade. The two sides are facing identical options in this trading scenario:

- Cooperate - Keep the trade agreement and allow the free flow of goods; or
- Defect - Break the trade agreement and impose tariffs on the foreign goods.

If both sides cooperate, each side earns a payoff of 15. If they both defect, each side earns a payoff of 10. If one side defects and the other side cooperates, the defector earns 25 while the cooperator earns 5.

(a) Write down the payoff matrix of this game.

Solution:

King Robert / Free Cities	*Cooperate*	*Defect*
Cooperate	*15, 15*	*5, 25*
Defect	*25, 5*	*10, 10*

(b) Does King Robert or the Free Cities have a dominant strategy?

Solution: Let's look at the situation from King Robert's perspective.

Case 1: suppose Free Cities cooperate.

- *If King Robert cooperates, then he gets a payoff of 15.*
- *If King Robert defects, then he gets a payoff of 25.*

So defecting is better because it results in higher payoff.

Case 2: suppose Free Cities defect.

- *If King Robert cooperates, then he gets a payoff of 5.*
- *If King Robert defects, then he gets a payoff of 10.*

So defecting is better because it results in higher payoff.

Consequently, no matter what Free Cities are doing, it's always optimal to defect. Recall that a dominant strategy is a strategy which is optimal no matter what your opponent does. Hence, defecting is a dominant strategy.

Notice that the payoffs for Free Cities are symmetric because if we switch King Robert and Free Cities in the payoff matrix above, nothing will change. Consequently, defecting is also a dominant strategy for Free Cities.

(c) Find maxmin strategy of King Robert.

Solution:

Case 1: suppose King Robert cooperates. To punish him (that is to give him the smallest payoff possible), Free Cities will defect. As a result, King will get 5.

Case 2: suppose King Robert defects. To punish him, Free Cities will defect. As a result, King will get 10.

That was ***min*** *part of* ***maxmin****.*

Now King Robert is trying to maximize his payoff by picking the case that will give him the maximum payoff - this is ***max*** *part of* ***maxmin****.*

As Case 2 yields highest payoff (10 vs 5), defecting is a maxmin strategy for King Robert.

Notice that it's not a coincidence that defecting is both dominant and maxmin strategy. In fact, any dominant strategy is maxmin! However, not every maxmin strategy is dominant.

(d) Find a Nash Equilibrium for this game.

Solution: Recall the definition: all players should play their best strategy given what competitors are doing.

Previously, we found that defecting is a dominant strategy for both players. The nice thing about dominant strategies is that we don't have to worry about what the opponent is doing - it's always the best choice. Therefore, the case when both players are defecting is a unique Nash equilibrium.

(e) Is the Nash equilibrium you found earlier Pareto optimal?

Solution: Recall the definition: an equilibrium is Pareto optimal if there is no way to make someone better off, without making others worse off.

In the Nash equilibrium, both players are getting 10. But if they were cooperating, both would get 15 and be better off! Hence, the Nash equilibrium is not Pareto optimal.

Exercise 2. Game in a Duopoly

SpaceX and Blue Origin are the only two firms who sell private trips to space. Market demand is given by the equation $Q_D = 32 - 2P$ (prices, in millions of \$)and consumers who buy do so at the firm with the lowest price. If SpaceX and Blue Origin charge the same price, half the buyers go to each firm. For the companies, providing each trip costs \$4 million and assume there are no fixed costs. Both firms set their price simultaneously.

(a) What is the joint-profit maximizing price (i.e. what price would SpaceX and Blue Origin charge if they were able to collude)? What profit would each company make if it set this price?

Solution: This is a collusion duopoly setting – where firms collude to act as a monopolist and maximize joint profits. We are given that $MC = 4$ *and we can get* $MR : P = 16 - Q$*. Setting* $MC = MR$*, we get*

$$4 = 16 - Q_M \implies Q_M = 12$$

Subbing into the demand, we also have $P_M = 16 - 0.5Q_M = \$10$ *mil.*

Thus, each firm offers 6 trips and prices it at \$10 mil.

Since there are no fixed costs ($FC = 0$*) and marginal costs are constant,*

$$TC = VC = MC \cdot Q = 4Q$$

Total profits for each firm $= TR - TC = \$10 * 6 - \$4 * 6 = \$36$ *mil.*

(b) Suppose SpaceX and Blue Origin compete by simultaneously choosing prices and can set either the joint maximizing price in part (a) or charge \$1 mil less. What are profits if both decide to charge \$1 mil less? Show a payoff matrix of the profits of the two firms.

Solution: If they decide to charge \$1 mil less, we can plug that price into the demand to get the quantity sold

$$Q_D = 32 - 2 * 9 = 14$$

If both firms set the price at \$9 mil, they split the market – sell 7 trips each and yield profits $= \$9 * 7 - \$4 * 7 = \$35$ *mil.*

If only one firm lowers the price and sets it at $9 mil, it grabs the whole market and singlehandedly sells 14 trips, which yields profits $= \$9 * 14 - \$4 * 14 = \$70$ *mil.*

Thus, we have a payoff matrix as follows:

SpaceX / Blue Origin	*Collude*	*Cheat*
Collude	*36, 36*	*0, 70*
Cheat	*70,0*	*35, 35*

(c) What price will each firm charge in the equilibrium for this game?

Solution: Let's look at the situation from SpaceX's perspective.

Case 1: suppose Blue Origin colludes.

- *If SpaceX colludes, then it gets a payoff of 36.*
- *If SpaceX cheats, then it gets a payoff of 70.*

So in this case, cheating is better because it results in higher payoff.

Case 2: suppose Blue Origin cheats.

- *If SpaceX colludes, then it gets a payoff of 0.*
- *If SpaceX cheats, then it gets a payoff of 35.*

Again, in this case, cheating is better because it results in a higher payoff.

Consequently, no matter what Blue Origin does, it is always optimal for SpaceX to cheat. Recall that a dominant strategy is a strategy which is optimal no matter what your opponent does. Hence, cheating is a dominant strategy.

Notice that since the payoffs for Blue Orign are symmetric, defecting is also a dominant strategy for Blue Origin.

Thus, both firms have an incentive to cheat. The NE outcome is when both firm cheat. Thus, firms make a profit of $35 mil in equilibrium. They will do so by charging a price of $9 mil.

Exercise 3. Monopolistic Competition

Use the following information to answer the next **two** questions.

The Sweet Shop sells ice cream in a monopolistically competitive market, and is currently realizing positive profits. It currently faces a demand curve where $P = 10 - \frac{1}{10}Q$, with $MC = \frac{1}{5}Q$.

1. How much will the Sweet Shop decide to produce, and what price will it sell at in the short run?

   Solution: A monopolistically competitive firm will decide on quantity by setting $MR = MC \Rightarrow 10 - \frac{1}{5}Q^* = \frac{1}{5}Q^*$*. This gives* $Q^* = 25$*, and going up to the demand curve, gives that* $P^* = \$7.5$*.*

2. Which of the following will occur in the long run?

   (a) Seeing profits, new firms that make identical products to the Sweet Shop will enter and sell at a slightly lower price, taking away all demand from the Sweet Shop.

   (b) Seeing profits, new firms producing similar products will enter the market, but the demand for the Sweet Shop, and thus the price and output decision, will remain the same.

   (c) Seeing profits, the Sweet Shop will produce more ice cream than in the short run but continue to sell at the same price.

   (d) Seeing profits, new firms will enter the market and cause demand for the Sweet Shop to shift to the left.

   Solution: (d). In the long run, other firms will enter and increase the number of substitutes available for Sweet Shop ice cream, resulting in demand for this particular firm shifting to the left. In equilibrium, profits will be equal to 0.

3. Suppose Sweet Shop's total cost curve is $\frac{1}{10}Q^2$. In the long-run, if Sweet Shop is producing $Q = 20$, what price must it be charging?

   Solution: In the long-run, we know that the monopolistically competitive firm makes zero profits, so $P = AC = TC/Q$*. Given TC above,* $P = \frac{1}{10} * 20 = \$2$*.*

Multiple choice questions

Airline Pricing Competition. Two firms are competing in the airline industry. Consider the following game matrix representing their profits in millions of dollars.

Firm A/ Firm B	Raise Price	Don't Raise Price
Raise Price	50,50	X,100
Don't Raise Price	100,X	60,60

1. Suppose X = 70. What is firm A's maxmin strategy? What is firm B's maxmin strategy?

    a. Firm A: Raise; Firm B: Raise

    b. Firm A: Raise; Firm B: Don't Raise

    c. Firm A: Don't Raise; Firm B: Raise

    d. Firm A: Don't Raise; Firm B: Don't Raise

    Solution: (d). To determine Firm A's maxmin strategy: Suppose Firm B is going to punish Firm A and correctly anticipates what Firm A will play. If Firm A plays R (Raise the price), Firm B will punish A by playing R (which gives Firm A a payoff of 50 instead of 70). If A plays D (Don't raise the price), Firm B will punish Firm A by playing D (gives Firm A a payoff of 60 instead of 100. Firm A wants to choose the max of these two options (getting 50 or getting 60). Obviously getting 60 from playing D is better, so Firm A's maxmin strategy is D (Don't raise the price). By symmetry, we see that Firm B's maxmin strategy is the same. Therefore, the maxmin strategies for both firms are D (Don't raise the price).

2. How many Nash equilibrium/a does this game have with X = 70?

    a. 0

    b. 1

    c. 2

    d. 3

    e. 4

    Solution: (c) There are two Nash Equilibria. The first is when A raises the price and B doesn't. The second is when A doesn't raise the price and B does. In either *case, to check if a set of strategies is a Nash Equilibrium, we ask if a player wants to deviate from his Nash Equilibrium strategy, holding fixed that his opponent is also playing his Nash Equilibrium strategy. If neither player wants to deviate, we have found a pair of Nash Equilibrium strategies.*

3. Now suppose X = 40. What is firm A's maxmin strategy? What is firm B's maxmin strategy?

   a. Firm A: Raise; Firm B: Raise

   b. Firm A: Raise; Firm B: Don't Raise

   c. Firm A: Don't Raise; Firm B: Raise

   d. Firm A: Don't Raise; Firm B: Don't Raise

   Solution: (d). We follow the same logic as before. Let's consider it from B's perspective this time. Suppose that Firm A correctly anticipates what B will play and chooses to punish him. If B plays R, then A will punish him by playing D(which gives Firm B a payoff of 40 instead of 50). If B plays D, then A will punish him by playing D as well (which gives Firm B a payoff of 60 instead of 100). Now, B has to decide whether he is better off paying R or D. Since Firm B gets a higher payoff playing D (60 versus 40), that is his maxmin strategy. By similar logic, A's maxmin strategy is also D. Thus, both players maxmin strategy is Don't raise the price (D).

4. How many Nash equilibrium/a does this game have with X = 40?

   a. 0

   b. 1

   c. 2

   d. 3

   e. 4

   Solution: (b). Now we find just one Nash Equilibrium, given by both firms deciding not to raise the price (D). Note that it is a dominant strategy for both players to play D.