Discussion 12

1 Topics

- First and third degree price discrimination
- Oligopoly: Collusion and Cournot Duopoly

2 Review: Price Discrimination

- **First degree PD:** monopolist knows the willingness to pay (WTP) of each consumer and charges consumers prices equal to their WTP; as a result, CS equals zero.
  - When $MC = AC$ is constant, monopolist sets prices such that the demand curve acts as the MR and produces Q_{PC} – which is the efficient outcome. Hence, there is no DWL.
  - In this case, assuming linear demand, the total profits by a monopolist is given by the area of the triangle that lies between the demand and the MC curves.
- **Third degree PD:** monopolist charges different groups of consumers different prices.
  - **Problem-solving steps:** separate demand equations are given for different groups of consumers. You follow the same profit maximization steps as for a monopolist, but separately for each consumer group, i.e. for each group, you first set $MR = MC$ to get Q_M, then use the corresponding demand to get P_M.

3 Review: Oligopoly

- **General Setting:** industry with few firms; firms have some market power i.e. can influence prices. Products can be homogenous or differentiated. Firms exhibit *strategic behavior* i.e. production decisions of a firm depends on behavior of other firms.
- **Collusion model:** firms collude to act as a monopolist – they jointly decide to produce monopolist quantities and charge monopolist prices because they want to maximize joint profits. They split the profits. Participating firms have incentive to cheat.
- **Cournot model :** a model of quantity competition
  - Setting: two firms; each takes the others' output as given; goal: maximize profits.
  - To get equilibrium quantity and prices, we build reaction functions.
  - Given linear demand and constant MC, reaction functions are *linear* and can be pinned down by *two* points: a) if the other firm produces Q_{PC}, you react by producing zero output; b) if the other firm produces zero output, you react by producing Q_M – acting like a monopolist. The resulting reaction function for firm 1 takes the form: $q_1 = Q_M - \frac{1}{2}q_2$ and for firm 2: $q_2 = Q_M - \frac{1}{2}q_1$.

- Substitute one reaction function into another to get q_1^* and q_2^*.
- Important Result: if both firms face the *same, constant* MC, then $q_1^* = q_2^* = \frac{2}{3}Q_M$.

4 Exercises

Exercise 1. First degree price discrimination
Suppose a monopoly is a first-degree price discriminator in the market for cell phones. Cell phone demand is given by $Q = 100 - P$. The monopoly's marginal cost is $MC = 20$.

1. What are the monopoly's profits if it practices first degree price discrimination? Are these higher or lower than when the monopoly cannot price discriminate?

   Solution: Under first degree price discrimination, the monopoly charges every consumer exactly what they are willing to pay. Hence it is as if his marginal revenue curve is the demand curve itself. Thus setting $MR = P = MC$, we solve $100 - Q = 20$, so $Q = 80$ is the quantity the monopolist sells. The monopolist's profits are thus the area of a large triangle: $\frac{1}{2}(\$100 - \$20)(80) = \$3200$.
   If the monopoly cannot price discriminate, then its marginal revenue curve is $MR = 100 - 2Q$. Setting $MR = MC$, we have $100 - 2Q = 20$, so $Q = 40$. Plugging this into the demand curve we have $P = 60$. Profits are therefore $Q(P - AC) = 40(\$60 - \$20) = \$1600$. Recall that if MC is constant, $MC = AC$.
   Note that in general first degree price discrimination will result in higher profits than when the firm can't discriminate

2. What is consumer surplus in this market when the monopoly practices first degree price discrimination? Is this higher or lower compared to the situation where the monopoly cannot price discriminate?

   Solution: Under first degree price discrimination, consumers have to pay exactly what they are willing to pay– therefore, consumer surplus is exactly equal to zero in this case. When the monopoly cannot price discriminate but instead charges the uniform price of $P = \$60$ and sells $Q = 40$ units as we found in part 1, consumer surplus is equal to $\frac{1}{2}(\$100 - 60)(40) = \$800$.

3. What is the dead weight loss in this market when the monopoly practices first degree price discrimination? Is this higher or lower than when the monopoly cannot price discriminate?

   Solution: In a perfectly competitive market when the quantity sold is such that $P = MC$, deadweight loss is zero – it is the efficient level of output. Note that when the monopoly practices first-degree price discrimination, it produces exactly this same quantity where $P = MC$. So, there is no deadweight loss when the monopoly practices first degree price discrimination.
   When the monopoly cannot practice first degree price discrimination, it produces a

smaller quantity.
These lost units generate a deadweight loss of $\frac{1}{2} \cdot (\$60 - \$20) \cdot (40) = \$800$.

Exercise 2. Third degree price discrimination
Suppose there is only one airline to serve a certain local airport. The airline serves both students and the general public. The airline's marginal cost is given by $MC = 20$. Suppose the student demand is given by $P = 120 - Q$ and the general public demand is given by $P = 200 - Q/4$. What price should the airline charge each group for tickets? What is this monopolist's total profit?
Solution: We solve the monopoly problem separately for each group.
Students: $P = 120 - Q \longrightarrow MR = 120 - 2Q$. *Setting* $MR = MC$, *we get* $120 - 2Q = 20 \longrightarrow Q = 50$. *Plug 50 into the demand function to get* $P = \$70$.
General Public: $MR = MC$ *so* $200 - Q/2 = 20 \longrightarrow Q = 360$. *Plug* $Q = 360$ *into* $P = 200 - Q/4$ *to get* $P = \$110$.
Profit from sales to students: $50 \cdot (\$70 - \$20) = 50 \cdot 50 = 2500$.
Profit from sales to general public: $360 \cdot (\$110 - \$20) = 360 \cdot \$90 = \$32,400$.
Total profit: $\$2500 + \$32400 = \$34900$.

Exercise 3. Collusion
Suppose there are 3 countries that are the only countries in the world to produce oil. The market demand for oil is $P = 260 - 2Q$. Marginal cost is \$20 on each unit sold.

a) Suppose they choose to collude. They first figure out how to maximize total profits and then divide the production and profits equally between them. How much do they each produce in this scenario? What profits does each earn?

Solution: To maximize total profits, they must produce the quantity that a monopolist would produce between the 3 of them. Thus, solve the monopolist's problem: $MR = MC \leftrightarrow 260 - 4Q = 20$ *which gives* $Q = 60$ *and plugging this quantity back into the market demand, we get* $P = \$140$.
So, each firm produces $q_1 = q_2 = q_3 = 20$ *for a total revenue* $= \$140 * 20 = \$2800$.
TC for each is $20 * \$20 = \$400$ *so each earns profits* $= TR - TC = \$(2800 - 400) = \$2400$.

b) Suppose one of the 3 countries cheats and produces an extra 20 barrels. What are the profits of the 3 countries in this scenario? Does it pay to be the cheater?

Solution: Total market $Q = 80$ *so now* $P = \$100$.
*The country that cheated is making 40 units so its revenue is \$4000. Total costs = 40 * \$20 = \$800, so its profits when it cheats are \$3200.*
The other countries profits go down to $\$(100 - 20) * 20 = \$1600$.
Thus cheating benefits the cheater and hurts everyone else.

c) Now suppose all 3 countries cheat and produce the extra 20 barrels. What are the profits of each?

Solution: Market quantity $Q = 120$ then $P = \$20$.
Since this is the same as MC, all three countries are making zero profits!
If everyone cheats, everyone loses.

Exercise 4. Cournot Duopoly

Consider the market for central processing units (CPUs), a key component in modern computers. This market consists of two firms: Intel and AMD. For simplicity, assume that both Intel and AMD have identical cost structures, where $MC = AC = 30$ (we will change this later) for each firm. On any given day, the market demand for CPUs is given by $P = 120 - Q$.

a) Suppose the market for CPUs was controlled by a monopoly with the same cost structure as Intel and AMD. How many CPUs would this monopoly produce (call this Q_M), and what price would it charge P_M ?

Solution: Solving the monopolist's problem in the usual way yields

$$MR = MC \leftrightarrow 120 - 2Q = 30 \Rightarrow Q_M = 45.$$

Plugging into the demand function

$$P_M = \$75.$$

b) Suppose instead the market for CPUs was perfectly competitive, with every firm having the same cost structure as Intel and AMD. What would be the market equilibrium quantity Q_{PC} and price P_{PC}?

Solution: Solving the perfectly competitive firm's problem $P = MC = 30$,

$$120 - Q = 30 \Rightarrow Q_{PC} = 90.$$

c) Now return to reality, where Intel and AMD compete as Cournot duopolist. What is the reaction function of Intel? What is the reaction function of AMD?

Solution: The reaction function for Intel is $q_{Intel} = 45 - \frac{1}{2}q_{AMD}$ and $q_{AMD} = 45 - \frac{1}{2}q_{Intel}$.
The symmetry in the reaction functions comes because both firms have identical costs.
Recall that $Q_M = 45$, which gives away one intercept in the reaction function – you would produce at this level if you believed the other firm would produce zero units.
The other intercept of each firm's reaction function signifies how you would react if the other firm was producing at Q_{PC} – i.e. here, you would produce 0 units if the other firm was producing 90 units. Using these two points, a linear reaction function can be drawn.

d) Find the quantity produced by each firm in a Cournot equilibrium, q^*_{Intel} and q^*_{AMD}. Then find the market quantity Q_C and market price P_C under this Cournot duopoly.

Solution: Two methods to get this answer. The first method is the general one where we find the intersection of the firms reaction functions by substituting one function in another.

$$q_{Intel} = 45 - \frac{1}{2}\left(45 - \frac{1}{2}q_{Intel}\right) \iff \frac{3}{4}q_{Intel} = \frac{45}{2} \iff q^*_{Intel} = 30 \implies q^*_{AMD} = 45 - \frac{1}{2}q^*_{Intel} = 30.$$

The second method makes use of the fact that since both firms have the same costs, they maximize profits the same way. Hence, they produce equal quantities in equilibrium. Thus, we can set $q^*_{Intel} = q^*_{AMD}$. This gives us

$$q^*_{Intel} = 45 - \frac{1}{2}q^*_{Intel} \iff \frac{3}{2}q^*_{Intel} = 45 \iff q^*_{Intel} = q^*_{AMD} = 30$$

Tip: Under this Cournot duopoly setting with same constant marginal costs for firms, you can directly set $q^*_{Intel} = q^*_{AMD} = \frac{2}{3}Q_M = \frac{2}{3}45 = 30$, i.e. firms produce quantities equal to two-thirds of the monopoly quantity.
In any case, we get, $Q_C = q^*_{Intel} + q^*_{AMD} = 60$ and $P_C = \$60$.

e) Compare the three industrial structures: monopoly, Cournot duopoly, and perfect competition. Rank these in terms of firms profits and the welfare of consumers (Hint:there is no need to calculate anything here. Use your intuition to rank these by comparing prices and quantities only.)

Solution: In terms of firms profits, profits for monopoly > profits for a firm in Cournot > profits for a perfectly competitive firm = 0, since marginal cost equals average cost and is constant.
In terms of consumer surplus, CS under perfect competition > CS under Cournot > CS under monopoly.

f) Suppose Intel's marginal cost is $MC = 20$. What's Intel's reaction function?

Solution: Given that $MC = 20$ if Intel were a monopoly then $MR = MC \leftrightarrow 120 - 2Q = 20 \leftrightarrow Q'_M = 50$.
On the other hand, if the firm were in a perfectly competitive market $AR = MC \leftrightarrow 120 - Q = 20 \leftrightarrow Q'_{PC} = 100$.
Thus, the reaction function is given by $q_{Intel} = 50 - \frac{1}{2}q_{AMD}$.
Again, to get this answer, verify what the intercepts of the reaction function signify.

g) Find the new quantity produced by each firm in a Cournot equilibrium, q^*_{Intel} and q^*_{AMD}. Can you use the $\frac{2}{3}$ rule?

Solution: There is no symmetry in the firms' costs anymore – so you cannot use the shortcut $\frac{2}{3}$ rule directly. The equilibrium quantities are not equal to $\frac{2}{3}$ the monopoly quantity.

Use the first method to get the quantities, i.e. substitute one reaction function into another.

$$q_{Intel} = 50 - \frac{1}{2}\left(45 - \frac{1}{2}q_{Intel}\right) \iff \frac{3}{4}q_{Intel} = \frac{55}{2} \iff q^*_{Intel} = \frac{110}{3}$$

Moreover, $q^*_{AMD} = 45 - \frac{1}{2}q^*_{Intel} = 45 - \frac{1}{2} \cdot \frac{110}{3} = \frac{80}{3}$.
Note that the firm with lower cost is producing a higher quantity. Total production in this case is higher than in the case with symmetric costs.

Exercise 5. Additional Exercise on Monopoly
Verisson is a monopoly in the provision of broadband plans in Madison. Verisson's variable cost and marginal cost are given by $VC = 4q$ and $MC = 4$, respectively. Verisson's fixed cost is equal to \$1,500 (it requires a huge investment in infrastructure to be able to provide broadband plans). The demand for broadband plans in Madison is given by $Q = 84 - P$.

a) Derive the TC for Verisson. Does Verisson's technology exhibit economies of scale?

Solution: $TC = FC + VC$. Then $TC = 1500 + 4q$.
Economies of scale refer to the observation that ATC is decreasing as Q increases (over a relevant range determined by demand). In this specific case, $MC = 4$ is constant, so $ATC = TC/q = 1500/q + 4$, which always decreases as q increases. Thus, Verisson's technology exhibits economies of scale.

b) Derive the Marginal Revenue (MR) of Verisson. Find the number of plans that Verisson provides and the price at which they sell them. Is Verisson making profits?

Solution: To compute MR, you have to first express the demand in terms of P.
So you get AR: P = 84 - Q. Using the fact that MR has the same y-intercept and a slope that is twice as steep if the demand is linear, you get that MR = 84 - 2Q.
Using the profit-maximizing condition MC = MR, Verisson would choose Q_M such that MC = MR.
Substituting the equations, 4 = 84 - 2Q. Then Q = 40. If Verisson only provides 40 plans, they can sell them at a price P =\$ 44 (plugging in Q into the demand curve).
Profits $= TR - TC = P * Q - 4 * Q - 1500 = \$44 * 40 - \$4 * 40 - \$1500 = \$100$. *Verisson is making positive economic profits.*

c) Compute the CS and PS in the market. What is the DWL of this monopoly?

Solution: Graphing MR, MC and D, it is easy to verify that
$CS = \frac{1}{2} * (\$84 - \$44) * (40 - 0) = \$800$,
$PS = (\$44 - \$4) * (40 - 0) = \$1,600$.
If the market was efficient (for example, in a perfectly competitive market) MC = P. This gives $Q_{PC} = 80$. *Therefore,* $DWL = \frac{1}{2} * (\$44 - \$4) * (80 - 40) = \$800$.

A rival firm Chartel, that has the same cost functions as Verisson, is considering whether to enter the market or not. If Chartel enters the market, demand is equally split between both firms. Thus, the demand for broadband plans that Chartel would face in Madison is given by $Q = 42 - 0.5P$.

a) Derive the MR for Chartel. Find the number of broadband plans that Chartel would offer and the price at which it would offer those plans if it decides to enter.

Solution: If Chartel decides to enter, then the demand it faces is half of the market demand. Then express P in terms of Q, we have the demand for Chartel as: $P = 84 - 2Q$, *so* $MR = 84 - 4Q$.
Profit maximizing requires $MR = MC$, *so you get that* $84 - 4Q = 4$, *so* $Q = 20$ *and* $P = 44$. *Profit:* $\Pi = TR - TC = 44 * 20 - 1500 - 4 * 20 = -700$.

b) Would Chartel enter the market of broadband plans in Madison?

Solution: Chartel would not enter the market because it would have negative profits. This exercise gives you intuition of why in some markets with very high fixed costs, it is "natural" that monopolies exist.

Multiple choice questions

1. Firms A and B are two firms in Cournot Duopoly. Market demand is $P = 50 - Q$ and $MC = 4$. How many units should firm B produce if firm A produces 20 units?

   a. 23
   b. 20
   c. 13
   d. 4

   Solution: If the market was perfectly competitive, total quantity produced would be obtained by setting: $P = MC \leftrightarrow 4 = 50 - Q \leftrightarrow Q_{PC} = 46$.
   If the market had a monopoly, total quantity produced would be obtained by setting: $MR = MC \leftrightarrow 50 - 2Q = 4 \leftrightarrow Q = 23$.
   Using these two points, we can obtain the reaction function of firm B is $q_B = 23 - \frac{1}{2}q_A$.
   So, if firm A produces 20, or $q_A = 20$, $q_B = 23 - \frac{20}{2} = 13$. *The correct answer is (c).*

2. A monopolistic firm faces a downward-sloping demand curve because:

   a. There are a large number of firms in the industry, all selling the same product.
   b. The demand for its product is always inelastic.
   c. Unlike a competitive firm, the amount a monopolistic firm sells affects the price of the good.
   d. Marginal revenue is negative throughout the feasible range of output.

   Solution: The answer is (c). In perfect competition, firms are small and their actions do not effect the equilibrium price. Perfectly competitive firms can also sell as many units as they want at this equilibrium price. This is not true for monopolists, who understand that as they increase the quantity they produce, the price that consumers are willing to pay decreases.

3. Which of the following would not be classified for a barrier to entry for a monopoly?

   a. patent laws

   b. economies of scale

   c. declining marginal revenue curve

   d. Government directives

   Solution: The answer is (c). Monopolies face a declining marginal revenue curve, but this is not something that causes a monopoly. The other three answers are all barriers to entry that help create monopolies.

4. If a monopolist can perfectly price discriminate, then

   a. It will charge just two different prices in two different markets.

   b. There will be no consumer surplus.

   c. The efficient quantity will not be produced.

   d. The deadweight loss is larger than if it cannot price discriminate.

   Solution: The answer is (b). With perfect price discrimination the monopolist charges each consumer the maximum amount they are willing to pay, so there is no surplus for any consumer.

5. Which of the following is true with third degree price discrimination?

   a. The monopolist earns higher profits than with first degree price discrimination.

   b. Consumer surplus is higher than with first degree price discrimination.

   c. The monopolist is able to charge a different price to each consumer.

   d. This is a profit maximizing strategy if resale is possible.

   Solution: The answer is (b). In perfect price discrimination, consumer surplus is zero. With third degree price discrimination, the monopolist can only charge different prices to different groups, and some people within each group will pay a lower price than their willingness to pay, so consumer surplus is positive.

6. Which of the following is **not** true when firms collude?

   a. Firms have an incentive to cheat.

   b. Firms choose price to maximize total profit.

   c. The quantity produced is the same as if there was a monopoly.

   d. Consumers surplus is higher than if there was a monopoly.

   Solution: The answer is (d). When firms collude, they maximize profit by setting the same price and quantity and price as a monopolist. Then consumer surplus will be the same as in the case of a monopolist.