Handout 1

Important Topics

- Math Review
- Opportunity Costs, Comparative Advantage and Absolute Advantage (if time permits)

Math Review

We'll use a lot of algebra and graphs in this course. It's important that the rote math steps are automatic so we can focus on the more interesting economic applications. So, you should be comfortable graphing equations, finding an equation from a graph, solving for two unknowns with two equations, and calculating percentages.

1. Graphing a line is usually most conveniently done if we first have the equation in *slope intercept* form.

   Slope intercept form: $y = mx + b$

   - m is the slope, rise over run.
   - b is the y-intercept, i.e. the point where the line given by the equation crosses the y-axis.

   Sometimes you instead encounter the same equation, but in the form $x = a + cy$. We have x in terms of y instead of in the usual slope intercept form. Here, it turns out a is the x-intercept and c is the inverse slope.

   Alternatively, you can simply transform the equation to slope-intercept form: $x = a + cy \leftrightarrow cy = x - a \leftrightarrow y = \frac{1}{c}x - \frac{a}{c}$. Identify the slope-intercept form and verify that the slope of the line is $\frac{1}{c}$ and the y-intercept is $-\frac{a}{c}$.

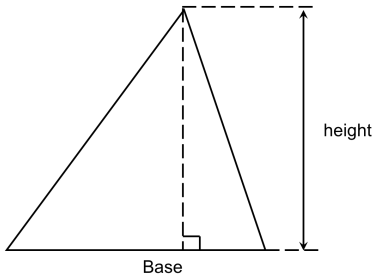
   To draw a line, it is sufficient to know two points on that line. Thus, if you are given a linear equation (i.e. an equation depicting a line), you can choose any two values of x-coordinates and substitute them for x in the equation to get corresponding y-coordinates, or vice versa, thereby obtaining two sets of coordinates on the line. If a given linear equation is in slope-intercept form, we can directly get a set of coordinates $(0, c)$ since the y-intercept (denoted by c) is known. In this case, we only need to find one other set of coordinates to draw the line.

   Conversely, we can find the equation of a graphed line by reading off two points on that line. We can denote the two points as (x_1, y_1) and (x_2, y_2). We can obtain the slope of the line as the ratio $m = \frac{y_2 - y_1}{x_2 - x_1}$. To get the equation of the line, we can then substitute coordinates in the equation: $y - y_1 = m(x - x_1)$.

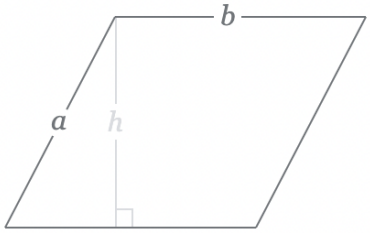
2. **Percentage change:** If we have an original and a new value, the percentage change is calculated as

$$\frac{New\ Value - Old\ Value}{Old\ Value} \times 100.$$

3. **Triangle area:** $= \frac{1}{2} \times base \times height$



4. **Parallelogram area:** $= base(b) \times height(h)$



Special cases:

- A rectangle is a parallelogram with all internal angles equal to 90^o. Same formula for area. $Area = b \times h$.
- A square is a rectangle with all sides equal. Same formula, but can be simplified. In this case, since $b = h$, we can use $Area = b^2$ or $= h^2$.

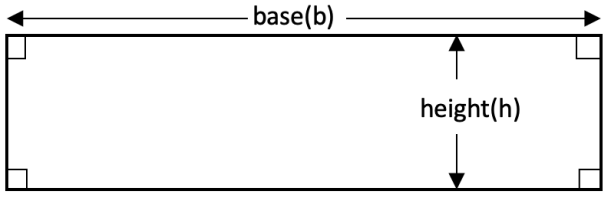


Figure 1: A rectangle

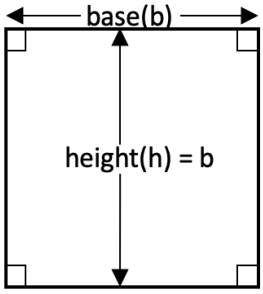
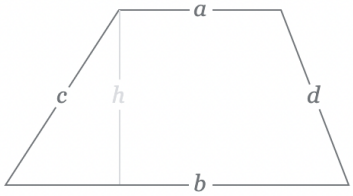
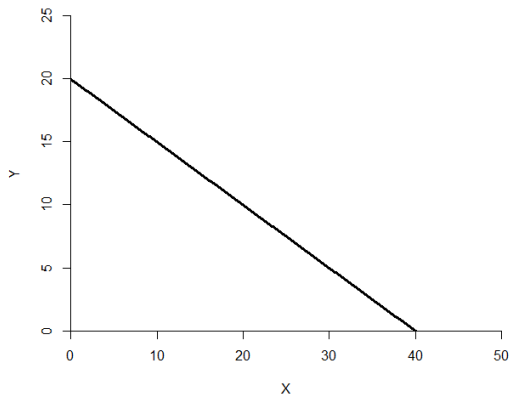


Figure 2: A square

5. (*Optional*) **Trapezium area:** $= \frac{1}{2} \times (a + b) \times h$



Exercise 1: Find the equation for the graph below. What is the area below the curve (the triangle formed with the axes)?



Solution: We see the y-intercept is 20. When we have both intercepts the slope will be $-\frac{y\text{-}int}{x\text{-}int}$. *So, the equation is* $y = 20 - \frac{1}{2}x$.

The area is $\frac{1}{2} base \times height = \frac{1}{2}40 \times 20 = 400$.

Exercise 2: Solve for x and y given the equations

$$y = 2x + 4y + 2$$

$$x = 2y + 4.$$

Solution: We can simplify the first equation to

$$-2x = 3y + 2$$

and then, dividing by negative two,

$$x = -\frac{3}{2}y - 1.$$

Now we can substitute this into the second equation,

$$x = -\frac{3}{2}y - 1 = 2y + 4.$$

Simplifying,

$$-5 = \frac{7}{2}y$$

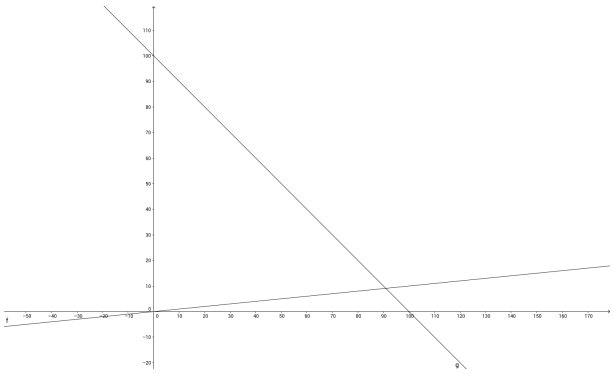
$$y = -10/7$$

. If $y = -10/7$, *we get* $x = -20/7 + 4 = \frac{8}{7}$.

Exercise 3: Graph $x = 10y$ and $y = 100 - x$. What is the area of the triangle enclosed by the two lines and the y-axis?

Solution:

You might rearrange $x = 10y$ *as* $y = \frac{1}{10}x$ *to observe that the y-intercept is at zero and the slope is* $\frac{1}{10}$*,*



To find the area of the triangle we need to find the base and height of the triangle. We can take the base as the y-axis section of the triangle, then base $= 100$*. To find the height we need the x coordinate where the two lines intersect. We substitute* $y = 100 - x$ *on* $x = 10y$*, to get*

$$x = 10(100 - x) = 1000 - 10x.$$

Simplifying,

$$x = \frac{1000}{11}.$$

Therefore height $= \frac{1000}{11}$ *and the area is* $\frac{1}{2}$*base* $\times$ *height* $= \frac{1}{2}\left(\frac{1000}{11} \times 100\right) = \frac{50000}{11}$*.*

Exercise 4: In 2013, Theranos, the health technology company, was valued at $9 billion. More recently, the company's value has been revised to $800 million. What is the percentage change in the company value?

Solution: Luckily, we can ignore all the zeroes that come along with billions and millions if we recognize that 800 *million is .8 of one billion. Therefore, the percentage change can be calculated as*

$$\frac{.8 - 9}{9} \times 100 = \frac{-8.2}{9} \times 100 = \frac{-820}{9}.$$

This reduces to $-91\frac{1}{9}\%$*. The company lost over* 91% *of its value.*

Exercise 5: On June 14, 2000, the Indiana Pacers lost to the Los Angeles Lakers in game four of the NBA finals, giving the Lakers a 3-1 edge on the series. Shaquille O'Neal made 10 free throws on 17 attempts. Reggie Miller made 11 free throws on 12 attempts. How many more free throws would Shaq have to make in a row to match Reggie's percentage?

Solution: Here, the challenge is mostly in setting up the correct problem.

After Shaq shoots x *more free throws, realize his percentage will be*

$$\frac{10 + x}{17 + x} \times 100.$$

Reggie Miller's percentage is fixed at

$$\frac{11}{12} \times 100$$

. We don't have to simplify these any further to solve, just set up the following inequality,

$$\frac{10+x}{17+x} \times 100 \geq \frac{11}{12} \times 100.$$

We can get rid of the 100s on both sides,

$$\frac{10+x}{17+x} \geq \frac{11}{12}.$$

Then cross-multiply,

$$120 + 12x \geq 187 + 11x$$

$$x \geq 67.$$

So Shaq needs to hit at least 67 *consecutive free throws to match Reggie Miller's percentage.*

Economics

Opportunity Cost: the best alternative that we give up, or forgo, when we make a choice or decision.

Comparative Advantage: the advantage that one (an individual or country) can produce a good or service at a lower opportunity cost than the other (individuals or countries)

Absolute Advantage: the advantage that one (an individual or country) can produce more of a good or service than the others (individuals or countries)

Exercise 6: A Robinson Crusoe Economy

Crusoe finds himself stranded on an island. He devotes 10 hours each day to either gathering coconuts or catching fish. Crusoe can gather 2 coconuts in an hour, but he needs 5 hours to catch a single fish. On the other side of the island is Friday. Friday also devotes 10 hours each day to gathering coconuts and fishing. He can gather 3 coconuts per hour and needs only 2.5 hours to catch a fish.

a.) What is the opportunity cost of gathering a coconut for Crusoe? For Friday? Who has the comparative advantage in gathering coconuts? Who has the absolute advantage?

Solution: See part b.)

b.) What is the opportunity cost of catching a fish for Crusoe? For Friday? Who has the comparative advantage in catching fish? Who has the absolute advantage?

Solution: One coconut costs Crusoe $\frac{1}{10}$ fish. One coconut costs Friday $\frac{2}{15}$ fish. One fish costs Crusoe 10 coconuts. One fish costs Friday 7.5 coconuts.

It may be helpful to organize this information in a table.

maximum output

	fish	*coconuts*
Crusoe	*2*	*20*
Friday	*4*	*30*

opportunity cost

	fish	*coconuts*
Crusoe	*10 coconuts*	$\frac{1}{10}$ *fish*
Friday	$\frac{15}{2}$ *coconuts*	$\frac{2}{15}$ *fish*

We can see that Friday has the absolute advantage in both fish and coconuts, since he can catch and gather a great quantity of both than Crusoe, in the same amount of time. Friday also has the comparative advantage in catching fish: The opportunity cost for catching 1 fish is 7.5 coconuts for Friday, but 10 for Crusoe. However, Crusoe has the comparative advantage in gathering coconuts: The opportunity cost for gathering 1 coconut is $\frac{1}{10}$ fish for Crusoe, less than $\frac{2}{15}$ which is Friday's opportunity cost.

Multiple Choice Questions

Exercise 7:

Suppose that the only input in the production of wheat and soy is labor. In 2017, Country A uses the same amount of labor in the production of wheat as it does in the production of soy, resulting in the production of 8,000 tons of wheat and 10,000 tons of soy. Suppose that in 2018, country A produces an additional 500 tons of wheat despite no increase in the total amount of labor available in the country. Assuming constant opportunity costs and holding everything else constant, how many tons of soy were produced in 2018?

a. 9375

b. 9600

c. 9750

d. 9900

Solution: First, it would be helpful to organize all the information in the question in one table:

	max. output 2017	*output 2018*	*OC*
wheat	*8000*	*8500*	$\frac{10}{8}$
soy	*10000*	*?*	$\frac{8}{10}$

Since the amount of labor and the opportunity costs are constant over the two years, in order to produce additional 500 tons of wheat, the country would have to decrease the produced quantity of soy. The country would have to reduce soy production by the opportunity cost of producing 500 tons of wheat: $500 * \frac{10}{8} = 625$. *Therefore, in 2018 the country will produce* $10,000 - 625 = 9,375$ *tons of soy. The correct answer is (1).*

Exercise 8:

Two firms, A and B, produce smartphones and tablets. In 4 days, firm A can produce 32 smartphones or 48 tablets. Firm B can produce 12 smartphones or X tablets in one day. For what value of X will firm B have the comparative advantage in producing tablets?

a. $X > 8$

b. $X < 8$

c. $X > 18$

d. $X < 18$

Solution: Lets write down what we know in a table:

	max. output		*OC*	
	A	*B*	*A*	*B*
smartphones	*32*	*12*	$\frac{3}{2}$	$\frac{X}{12}$
tablets	*48*	*X*	$\frac{2}{3}$	$\frac{12}{X}$

Firm B will have the comparative advantage in producing tablets when the opportunity cost for firm B of producing tablets is less than that of firm A:

$$\frac{2}{3} > \frac{12}{X}$$

$$2X > 36$$

$$X > 18$$

Which is option (3).