Extra Handout on Consumer Problem

1 Sketch of Solution for Quiz Question

Given the information, we want to first check if Kate currently satisfies the utility maximization rule. Since we are not given income, we can just assume that Kate is satisfying her budget constraint ($I = P_a \times A + P_o \times O$). Recall that this is utility maximization condition with two goods is:

$$MRS = \frac{MU_x}{MU_y} = \frac{P_x}{P_y}$$

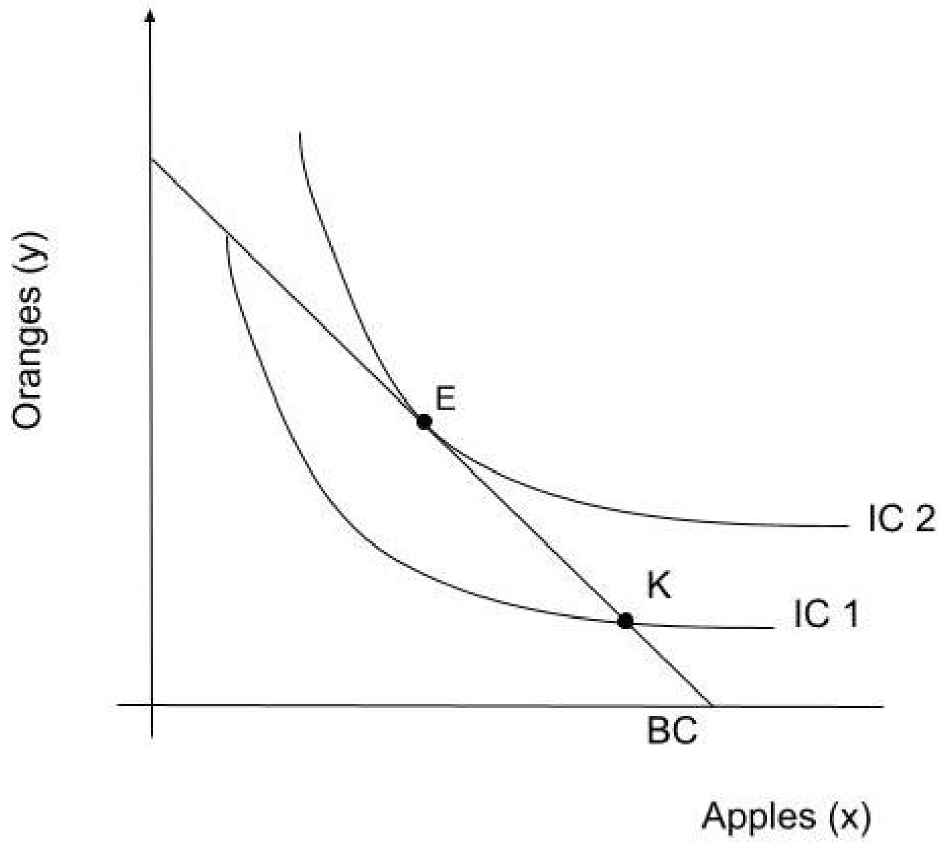
For the left side of the equation, we are given ratio of the marginal utility of apples to the marginal utility of oranges, i.e. $\frac{MU_a}{MU_o} = \frac{2}{3}$. Note that this notation is slightly different from our usual $\frac{MU_x}{MU_y}$, so we are effectively treating x = apples and y = oranges. For the right side of the equation, we are given the prices of apples $P_a = 3$ and $P_o = 3$. So, plugging this information in, we can check to see

$$MRS = \frac{MU_a}{MU_o} < \frac{P_a}{P_o}$$
$$MRS = \frac{2}{3} < \frac{3}{3}$$

In words, this means the MRS ($\frac{MU_a}{MU_o}$) is smaller than the price ratio ($\frac{P_a}{P_o}$). So, Kate is not at the optimal point.

Now we established that Kate is not at the optimum, what can she do to improve her situation? She can't change prices, since we assume individual consumers can't change the price. However, she can change her marginal rate of substitutions, because her marginal utilities are functions of how many apples or oranges she consumes. Specifically, we assume diminishing marginal utility, which means if she consumes more of one good, the marginal utility of that good decreases.

To get to the optimum, Kate needs $\frac{MU_a}{MU_o}$ to equal $\frac{P_a}{P_o}$. At the current point, $\frac{MU_a}{MU_o}$ is too small, so MU_a is too small and MU_o is too big. Since marginal utility has an inverse relationship with quantity consumed, this means Kate is consuming too many apples and too few oranges. Therefore, Kate needs to consume more oranges and fewer apples to get to the optimal point. The plot below shows Kate's current point, optimal point (equilibrium), and the related budget constraints and indifference curves.



A few notes on the plot:

- **BC (Budget constraint):** We don't know Kate's income, but we know that the slope of the budget constraint is $-\frac{P_x}{P_y} = -\frac{P_a}{P_o} = -\frac{3}{3} = -1$. So, just draw a downward sloping, straight line with the slope of -1 for the BC.
- **K (Kate's current point) and IC 1:** We know that the slope of the IC (indifference curve) is $-MRS = -\frac{MU_a}{MU_o} = -\frac{2}{3}$ at Kate's current point of consumption. This means the the IC is flatter than the BC (slope of BC = -1) at this point. We assume that Kate is still spending all of her income, so we find the point K at the intersection of the BC and IC 1, where the slope of the IC 1 is smaller than the slope of the BC.
- **E (Equilibrium point) and IC 2:** We know that at the equilibrium, $\frac{MU_a}{MU_o} = \frac{P_a}{P_o}$, which means the slope of the IC must equal to the BC at that point. Also, Kate should be maximizing her income (spending all of it), so E must be a point on both the BC and IC 2. We call this the point where the BC to "tangent" to IC 2.

2 Practice Problems (Midterm 2 Review Questions)

If you would like more practice on the consumer theory unit, try **questions 1 - 53** on the "midterm 2 review questions" packet (posted on Canvas under "Midterm 2").

Below are **some selected questions solutions** (I tried to focus on more tricky questions):

Question 10:

Cameras are a normal good. Therefore the Engel curve for cameras has a (blank) slope.

(a) positive

(b) negative

Answer: The correct answer is (a), because the Engel curve plots the relationship between income (on x-axis) and quantity demanded by consumers (on y-axis). For normal goods, when income increases, quantity demanded increases, so the slope of the Engel curve must be positive.

Question 14:

Using the model of consumer theory in which a consumer spends his income on two types of goods, which of the following statements are true?

- Statement (I): If both prices double but income is unchanged, the substitution effect must be zero.
- Statement (II): If income doubles and both prices also double, the optimal consumption bundle will not change.

(a) Statement (I) is true, but statement (II) is false.

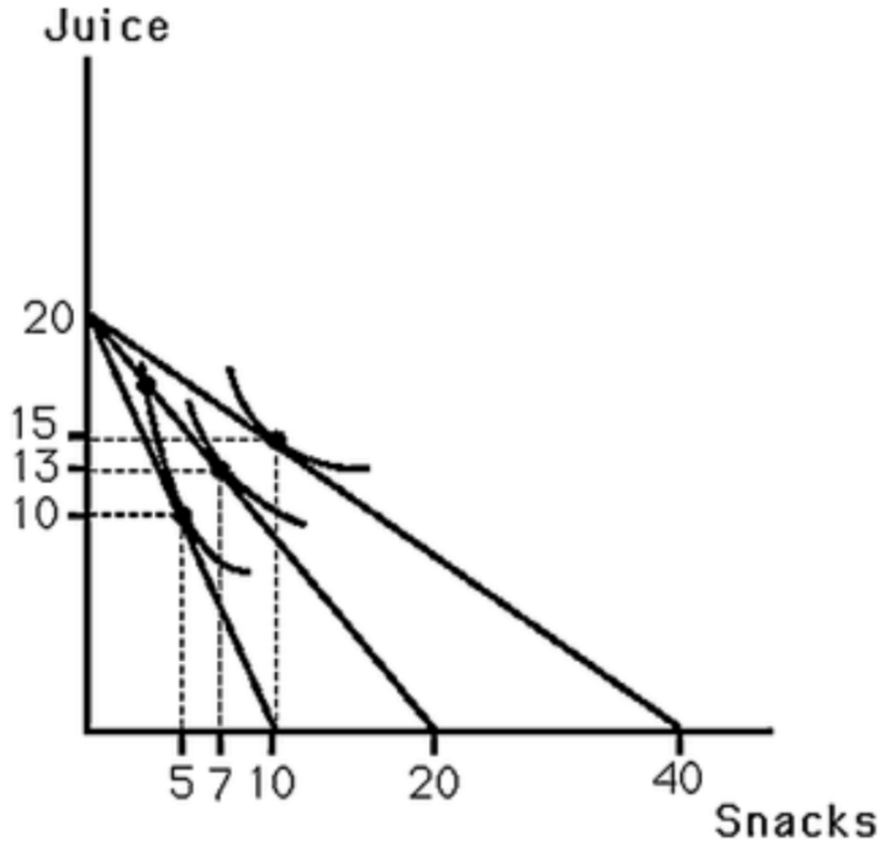
(b) Statement (I) is false, but statement (II) is true.

(c) Both statements are false.

(d) Both statements are true.

Answer: The correct answer is (d). Statement (I) is true because substitution effect is the change in consumption when the price ratio ($\frac{P_x}{P_y}$) changes. Notice that if both prices double, $\frac{2P_x}{2P_y} = \frac{P_x}{P_y}$, so there is no change to the price ratio. Statement (II) is correct, because the budget constraint will not shift when both income and prices double. Recall that the x-intercept of the budget constraint is $\frac{I}{P_x}$, so if income and price of x doubles, $\frac{2I}{2P_x} = \frac{I}{P_x}$. Thus the x-intercept is unchanged. The same applies to the y-intercept. Since the price ratio does not affect the indifference curve, we only need to consider possible changes to the budget constraint.

Question 23 (Similar to Discussion Section 6 Handout):



The figure above shows Lily's indifference curves for juice and snacks. Also shown are three budget lines resulting from different prices for snacks, assuming she has $20 to spend on these goods. Which of the following points are on Lily's demand curve for snacks? (Assume P represents the price of snacks and Q represents her quantity of snacks demanded.)

(a) P = $2, Q = 7

(b) P = $2, Q = 10

(c) P = $0.5, Q = 7

(d) P = $0.5, Q = 10

Answer: The correct answer is (d). Remember that the demand curve gives us the relationship between price of a good and the quantities demanded. Using the plot above, we can find the optimal consumption of snacks for Lily at each price. Recall that the x-intercept of the budget constraint is $\frac{I}{P_{snacks}}$, so given Lily's income ($40) and the x-intercept, we can find the price of snacks i.e. P_{snacks}. Start with the outer most budget constraint: given x-intercept = 40, $P_{snacks} = 20/40 = 0.5$. The optimal point of snack consumption is the point where the indifference curve is tangent to the current budget constraint. Looking at this point, Lily would consume 10 snacks when $P_{snacks} = 0.5$.

Question 24:

Julia only consumes bread and milk. For her, $MU_{bread} = \frac{1}{x}$ and $MU_{milk} = \frac{1}{y}$. Her income is $120 and prices of bread and milk are respectively $1 and $2. Then her equilibrium consumption bundle is

(a) 40 loaves of bread and 30 glasses of milk

(b) 20 loaves of bread and 50 glasses of milk

(c) 90 loaves of bread and 5 glasses of milk

(d) 60 loaves of bread and 30 glasses of milk

Answer: The correct answer is (d). To solve this problem, we need to use the two equations for utility maximization, the budget constraint and the utility maximization rule. Note, the question doesn't make this very clear, but we assume $x = Q_{bread}$ and $y = Q_{milk}$. We can plug the information that we are given, i.e. the prices and income into the budget constraint:

$$I = P_{bread} \times Q_{bread} + P_{milk} \times Q_{milk}$$
$$120 = 1 \times Q_{bread} + 2 \times Q_{milk}$$

We can also plug in the marginal utilities and prices into the utility maximization rule:

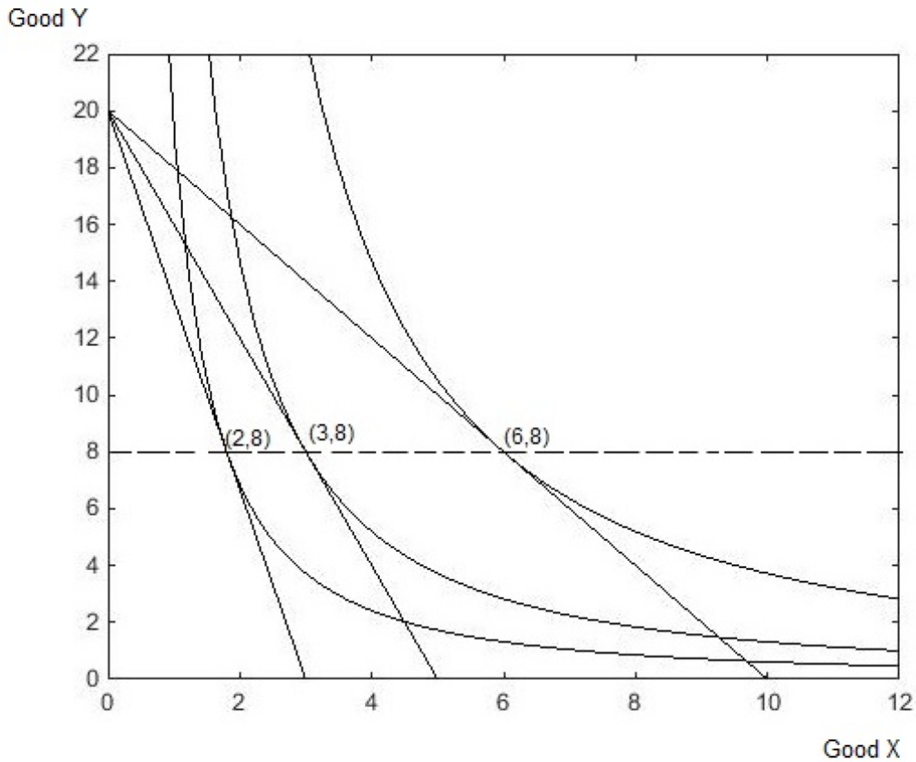
$$\frac{MU_{bread}}{MU_{milk}} = \frac{P_{bread}}{P_{milk}}$$
$$\frac{1/x}{1/y} = \frac{1}{2}$$
$$\frac{y}{x} = \frac{1}{2}$$
$$y = \frac{x}{2}$$

Recall that $x = Q_{bread}$ and $y = Q_{milk}$. So, we have $Q_{milk} = \frac{Q_{bread}}{2}$. We can then plug this into the budget constraint:

$$120 = 1 \times Q_{bread} + 2 \times \frac{Q_{bread}}{2}$$
$$120 = 2Q_{bread}$$
$$Q_{bread} = 60$$

Finally, using the previous result, $Q_{milk} = \frac{Q_{bread}}{2} = \frac{60}{2} = 30$.

Question 31 Martha has an income of \$20 and price of good Y is \$1. What can you tell



about her preferences for good X from this graph?

(a) Good X is an inferior good.

(b) At a price of \$2, she demands 6 units of good X.

(c) Demand for good X is perfectly inelastic.

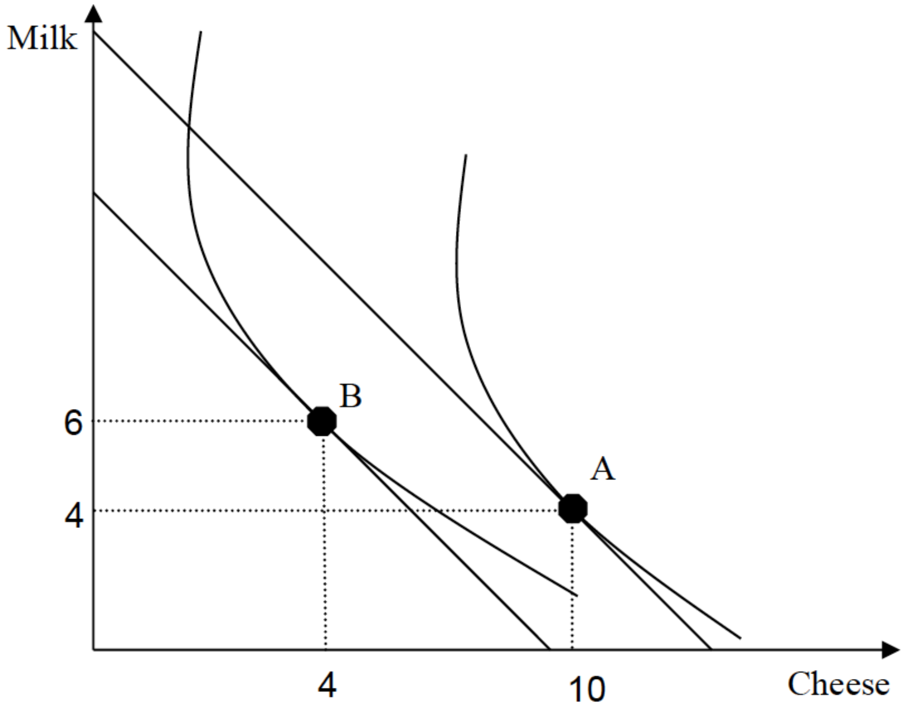
(d) Both a and b are true.

Answer: The correct answer is (b). For the outer most budget constraint, the x-intercept is $10 = Income/P_X$. So, given income \$20, $P_X = 20/10 = 2$. So, when $P_X = 2$, the corresponding budget constraint is tangent to the indifference curve at (6, 8). That is the optimal consumption bundle, so Martha demands 6 of good X at price of 2.

Option (a) is wrong: notice that as the price of X increases, the budget line pivots from x-intercepts of 10 to 5 to 3. Effectively, income decreases (since you can purchase less when prices increase), and we see that the optimal quantity of good X decreases (6 to 3 to 2). Thus good X is normal. Option (c) is wrong because perfectly inelastic demand means quantity demanded does not change for good X no matter how the price of good X changes.

Question 32 - 34

Use the information below for the next three questions: You are given the following indifference curves and budget lines for Tim. The prices of cheese and milk are fixed.



Question 32: If Tim's income goes down, his optimal consumption changes from point A to point B. Which of the following statements is true?

(a) Milk is a normal good for Tim.

(b) Cheese is an inferior good for Tim.

(c) Milk is a luxury good for Tim.

(d) Milk is an inferior good for Tim.

Answer: The correct answer is (d). Notice that when Tim's income goes down, it means his budget constraint shifts downward (in a parallel manner) from the outer to the inner budget constraint. Since there are not changes to the price ratio (or the slopes of the budget constraints remain the same), there is no substitution effect. When we move from the old equilibrium A to the new equilibrium B (reflecting decrease in income), optimal cheese consumption decreases from 10 to 4, optimal milk consumption increases from 4 to 6. Thus, cheese is a normal good and milk is an inferior good.

Question 33: Tim's income is $12 when he consumes at A, and is $7 when he consumes at B. What are the prices of cheese and milk, respectively?

(a) 1, 1/2

(b) 1, 2

(c) 2, 1/2

(d) 2, 2

Answer: The correct answer is (a). We can use the budget constraints to solve this, given the income (e.g. $12 at A) and optimal bundle of cheese and milk (e.g. 10, 4 at A).

$$I = P_{cheese} \times Q_{cheese} + P_{milk} + Q_{milk}$$
$$12 = P_{cheese} \times 10 + P_{milk} \times 4$$

We can do the same for point B:

$$I = P_{cheese} \times Q_{cheese} + P_{milk} + Q_{milk}$$
$$7 = P_{cheese} \times 4 + P_{milk} \times 6$$

Then we can solve for P_{cheese} and P_{milk} using the two budget constraints (BCs). You can solve for two unknowns (P_{cheese} and P_{milk}) in two ways. Both ways work, you can just pick the one you like!

Method 1: Isolate either P_{cheese} or P_{milk} using one of the BCs/equations, then plug into the other BC/equaiton to solve. For example, isolate P_{cheese} in the BC for point B. Then, plug the isolated equation for P_{cheese} into the BC for point A, and solve.

Method 2: At point A, income is 12 and at point B, income is 7. If we add 5 to 7, we get 12! So if we add 5 to both sides of the BC at point B, then we can set the two BCs equal to each other and solve. I show you how to implement method 2 below:

$$\begin{aligned}
P_{cheese} \times 10 + P_{milk} \times 4 = 12 = 7 + 5 &= P_{cheese} \times 4 + P_{milk} \times 6 + 5 \\
P_{cheese} \times (10 - 4) &= P_{milk} \times (6 - 4) + 5 \\
P_{cheese} \times 6 &= P_{milk} \times 2 + 5 \\
P_{cheese} &= \frac{P_{milk} \times 2 + 5}{6}
\end{aligned}$$

$$\begin{aligned}
7 &= P_{cheese} \times 4 + P_{milk} \times 6 \\
7 &= \frac{P_{milk} \times 2 + 5}{6} \times 4 + P_{milk} \times 6 \\
7 &= \frac{4P_{milk}}{3} + \frac{10}{3} + 6P_{milk} \\
7 - \frac{10}{3} &= \frac{4P_{milk}}{3} + \frac{18P_{milk}}{3} \\
\frac{11}{3} &= \frac{22P_{milk}}{3} \\
P_{milk} &= 1/2
\end{aligned}$$

$$\begin{aligned}
P_{cheese} &= \frac{P_{milk} \times 2 + 5}{6} \\
&= \frac{1/2 \times 2 + 5}{6} = 1
\end{aligned}$$

Question 34: What is his marginal rate of substitution at point A or point B?

(a) 1/2

(b) 1

(c) 2

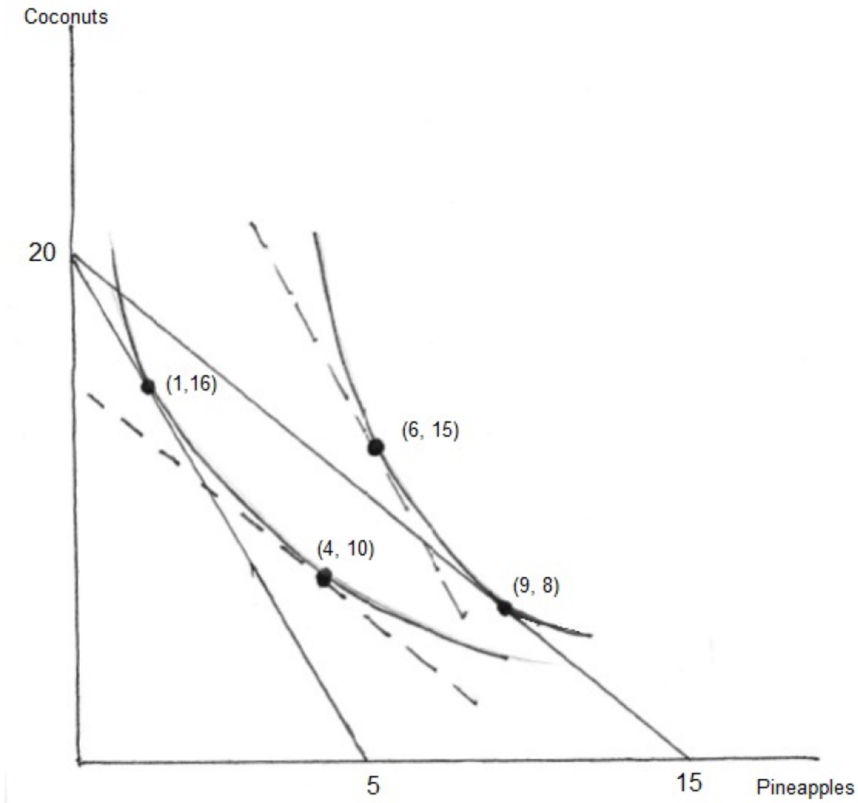
(d) 1/4

Answer: The correct answer is (c). Recall that at the point of optimal consumption, we are on the budget constraint, and the slope of the budget constraint is equal to the slope of the indifference curve. In equations, this means

$$\frac{MU_{cheese}}{MU_{milk}} = \frac{P_{cheese}}{P_{milk}}$$

Remember $MRS = \frac{MU_{cheese}}{MU_{milk}}$. Note: given the plot attached to this question puts cheese on the x-axis and milk on the y-axis, we treat x = cheese and y = milk. We always put MU_x in the numerator of the MRS. From Question 33, we got that $P_{milk} = 1/2$ and $P_{cheese} = 1$. Therefore, $\frac{P_{cheese}}{P_{milk}} = 1 \div 1/2 = 2$. Thus, $MRS = \frac{MU_{cheese}}{MU_{milk}} = 2$.

Questions 35



Question 35: What kind of goods are coconuts and pineapples, respectively?

(a) Normal; Giffen

(b) Inferior; Giffen

(c) Inferior; Normal

(d) Normal; Normal

Answer: The correct answer is (c). Consider the budget constraint (BC) with x-intercept = 5 as our starting point, and suppose price of pineapples decreases, so the budget constraint pivots out to where the x-intercept = 15. Finding the points where the BC is tangent to indifference curves, the initial equilibrium is (1, 16) (call this point E1), and the new equilibrium is (9, 8) (call this point E2).

Remember we can draw an intermediate budget constraint that reflects the changed price ratio but is still tangent to the old indifference curve (IC). The point where the intermediate budget constraint is tangent to the old IC is (4,10) (call this point E').

The income effect is identified when we go from point E' to point E2. In this case, we assume the price of pineapples decreases, so income effectively goes up (you can buy more with the same income when price decreases). Between E' and E2, notice that coconuts decreases from 10 to 8, and pineapples increases from 4 to 9. So, income effectively increases, and optimal coconut consumption drops, so coconuts are inferior goods. Optimal pineapple consumption increases, so it is a normal good.

Recall that normal goods are goods where when income goes up, demand goes up; inferior goods are goods where when income goes up, demand goes down; giffen goods are where when price goes up, demand goes up.